If origin, points $(2-x, 2, 2)$, $(2, 2-y, 2)$, $(2, 2, 2-z)$ are co-planar then prove that

$\frac{2}{x} + \frac{2}{y} + \frac{2}{z} = 1$

if origin $(0,0,0)$, $(2-x, 2, 2)$, $(2, 2-y, 2)$, $(2, 2, 2-z)$ are co-planar then

$$\begin{vmatrix} 2-x-0 & 2-0 & 2-0 \\ 2-0 & 2-y-0 & 2-0 \\ 2-0 & 2-0 & 2-z-0 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 2-x & 2 & 2 \\ 2 & 2-y & 2 \\ 2 & 2 & 2-z \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} -x & y & 0 \\ 0 & -y & z \\ 2 & 2 & 2-z \end{vmatrix} = 0$$

$$\Rightarrow -x\{-y(2-z) - 2z\} - y\{0 - 2z\} = 0$$

$$\Rightarrow 2xy - xyz + 2zx + 2yz = 0$$

$$\Rightarrow 2xy + 2yz + 2zx = xyz$$

$$\Rightarrow \frac{2xy + 2yz + 2zx}{xyz} = 1$$

$$\Rightarrow \frac{2}{z} + \frac{2}{x} + \frac{2}{y} = 1$$

$$\Rightarrow \frac{2}{x} + \frac{2}{y} + \frac{2}{z} = 1$$ (Proved).

A variable plane which is at a constant distance 3p from the origin O cuts the axes at L, M, N. Show that the locus of the point of intersection of the planes through L, M, N drawn parallel to the co-ordinate planes is $9(x^{-2}+y^{-2}+z^{-2}) = p^{-2}$

Let us assume the three planes passing through L, M, N intersect at P(α, β, γ).

equation of plane parallel to xy plane, yz plane, zx plane and passes through P are,

$z = \gamma$

$x = \alpha$

and $y = \beta$ respectively.

These planes are cutting z axis, x axis, y axis at $\overset{N}{(0,0,\gamma)}$, $\overset{L}{(\alpha,0,0)}$ and $\overset{M}{(0,\beta,0)}$ respectively.

The equation of plane which cuts the axes at L, M, N is

$$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1 \quad \text{——(i)}$$

Since distance of plane (i) from origin is 3p unit, hence,

$$\frac{\left|\frac{0}{\alpha}+\frac{0}{\beta}+\frac{0}{\gamma}-1\right|}{\sqrt{\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2+\left(\frac{1}{\gamma}\right)^2}} = 3p$$

$$\Rightarrow \frac{1}{\sqrt{\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}}} = 3p$$

$$\Rightarrow \frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2} = \frac{1}{9p^2}$$

$$\Rightarrow 9(\alpha^{-2}+\beta^{-2}+\gamma^{-2}) = p^{-2}$$

$\therefore$ Locus of (α,β,γ) is $9(x^{-2}+y^{-2}+z^{-2}) = p^{-2}$.

Prove that the equation of the plane which passes through point $(-1,3,2)$ and is perpendicular to the planes $x+2y+2z=5$ and $3x+3y+2z=8$ is $2x-4y+3z+8=0$

Equation of plane passes through $(-1,3,2)$ is $a(x+1)+b(y-3)+c(z-2)=0$ —(1)

Since plane (1) is perpendicular to $x+2y+2z=5$,

$$a+2b+2c=0 \quad \text{—(2)}$$

Since plane (1) is perpendicular to $3x+3y+2z=8$,

$$3a+3b+2c=0 \quad \text{—(3)}$$

from (2), (3)

$$\frac{a}{4-6}=\frac{-b}{2-6}=\frac{c}{3-6}=k \ (\neq 0 \text{ say})$$

$\Rightarrow a=-2k,\ b=+4k,\ c=-3k$

equation (1) becomes,

$$-2k(x+1)+4k(y-3)-3k(z-2)=0$$

$\Rightarrow 2x-4y+3z=\cancel{16}-8$

$\Rightarrow 2x-4y+3z+8=0$ (Proved)

A variable plane passes through the point (f,g,h) and meets the axes at L, M, N. If the planes through L, M, N and parallel to the axes meet at P, then prove that the locus of P is $\frac{f}{x}+\frac{g}{y}+\frac{h}{z}=1$

Let the co-ordinates of P is (α,β,γ)

Since planes through L, M, N and parallel to x axis, y axis, z axis meet at P therefore co-ordinates of L, M, N are $(\alpha,0,0)$, $(0,\beta,0)$, $(0,0,\gamma)$

Equation of plane which meets the axes at $L(\alpha,0,0)$, $M(0,\beta,0)$, $N(0,0,\gamma)$ is,

$$\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1 \quad \text{(intercept form) —(i)}$$

Since plane (i) passes through (f,g,h) eq (i) becomes $\frac{f}{\alpha}+\frac{g}{\beta}+\frac{h}{\gamma}=1$

So locus of $P(\alpha,\beta,\gamma)$ is $\frac{f}{x}+\frac{g}{y}+\frac{h}{z}=1$ (Proved).

Let $(2,3,-1)$ be the co-ordinates of the foot of the perpendicular drawn from the origin to a plane. Find the equation of that plane.

The perpendicular is drawn from origin $(0,0,0)$ and co-ordinates of its foot is $(2,3,-1)$.

So direction ratio of the perpendicular $2-0, 3-0, -1-0$ ie $2, 3, -1$

The plane is also passing through $(2,3,-1)$

So, equation of the plane $2(x-2)+3(y-3)-1(z+1)=0$

or, $2x+3y-z=14$.

Find the equation of the plane which passes through points $(1,1,2)$ and $(2,4,3)$ and perpendicular to the plane $x-3y+7z=6$,

Equation of the plane passes through $(1,1,2)$ is, $a(x-1)+b(y-1)+c(z-2)=0$ —(1)

Since this plane also passes through $(2,4,3)$ hence,

$a(2-1)+b(4-1)+c(3-2)=0$ —

$\Rightarrow a+3b+c=0$ —(2)

Since plane (1) and $x-3y+7z=6$ are perpendicular to each other so their normals are perpendicular to each other too.

hence, $a-3b+7c=0$ —(3)

from (2) and (3),

$$\frac{a}{21+3} = \frac{b}{1-7} = \frac{c}{-3-3} = k \ (\text{say } k \neq 0)$$

so, $a = 24k$, $b=-6k$, $c=-6k$

eq(1) becomes $24k(x-1)-6k(y-1)-6k(z-2)=0$ or, $4x-y-z=1$