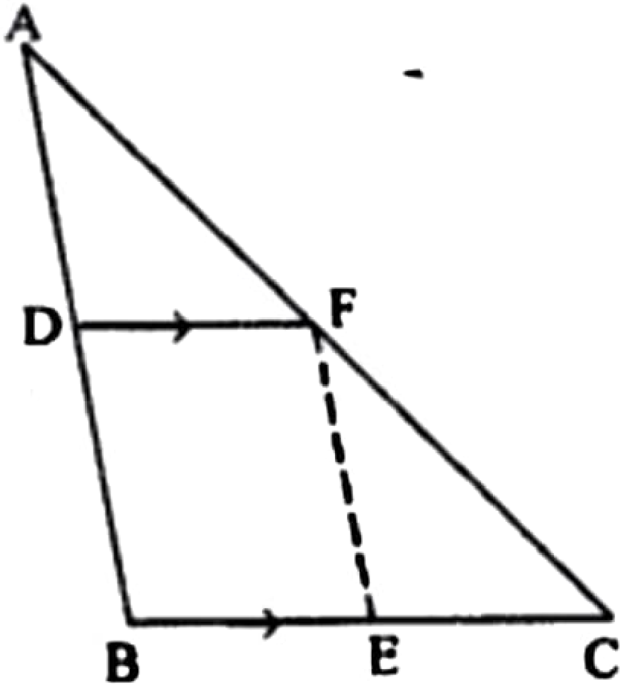
(*c*) **Given :** D and E are mid-points of AB, BC respectively and DF ∥ BC, AF = 2.6 c.m.

To prove : (*i*) BEF is a parallelogram

(*ii*) To calculate the value of AC



Proof : (*i*) In Δ ABC

∵ D is the mid-point of AB and DF ∥ BC

∴ F is the mid-point of AC ...(1)

Now, F and E are mid-point of AC and BC respectively.

∴ EF ∥ AB ....(2)

Now, DF ∥ BC

⇒ DF ∥ BE ....(3)

∵ EF ∥ AB [From (2)]

⇒ EF ∥ DB ....(4)

From (3) and (4), DBEF is a parallelogram

(*ii*) ∵ F is mid-point of AC

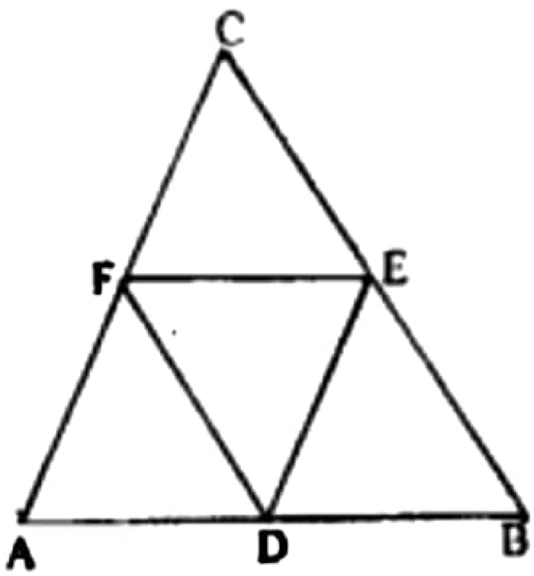
∴ AC = 2 × AF = 2 × 2.6 cm = 5.2 cm.

Prove that the four triangles formed by joining in pairs the mid-points of the sides c of a triangle are congruent to each other.

Solution:

Given: In Δ ABC, D, E and r,

F are mid-points of AB, BC and CA respectively. Join DE, EF and FD.



To prove :

$\Delta ADF \cong \Delta DBE \cong \Delta ECF \cong \Delta DEF$.

Proof : In ΔABC, D and E are mid-point of AB and BC respectively

$\therefore$ DE || AC or FC

Similarly, DF || EC

Similarly, DF || EC

$\therefore$ DECF is a parallelogram.

$\therefore$ Diagonal FE divides the parallelogram DECF in two congruent triangle DEF and CEF.

$\therefore \quad \Delta DEF \cong \Delta FCF$ ....(1)

Similarly we can prove that,

$\Delta DBE \cong \Delta DEF$ ...(2)

and $\Delta DEF \cong \Delta ADF$ ....(3)

From (1), (2) and (3),

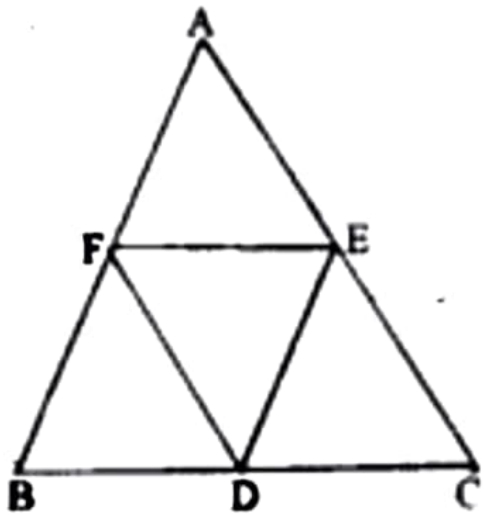
$\Delta ADF \cong \Delta DBE \cong \Delta ECF \cong \Delta DEF$

(Q.E.D.)

If D, E and F are mid-points of sides AB, BC and CA respectively of an isosceles triangle ABC, prove that ΔDEF is also F, isosceles.

Solution:

Given : ABC is an isosceles triangle in which AB = AC



D, E and F are mid point of the sides BC, CA and AB respectively D, E, F are joined

To prove : Δ DEF is an isosceles triangle.

Proof : D and E are the mid points of BC and AC

$\therefore$ DE ∥ AB and DE = $\frac{1}{2}$ AB ....(1)

Again, D and F are the mid-points of BC and AB respectively.

$\therefore \quad DF \parallel AC$ and $DF = \frac{1}{2} AC$ ....(2)

$\because \quad AB = BC$ (given)

$\therefore \quad DE = DF$

$\therefore \quad \Delta DEF$ is an isosceles trianlge

(Q.E.D.)

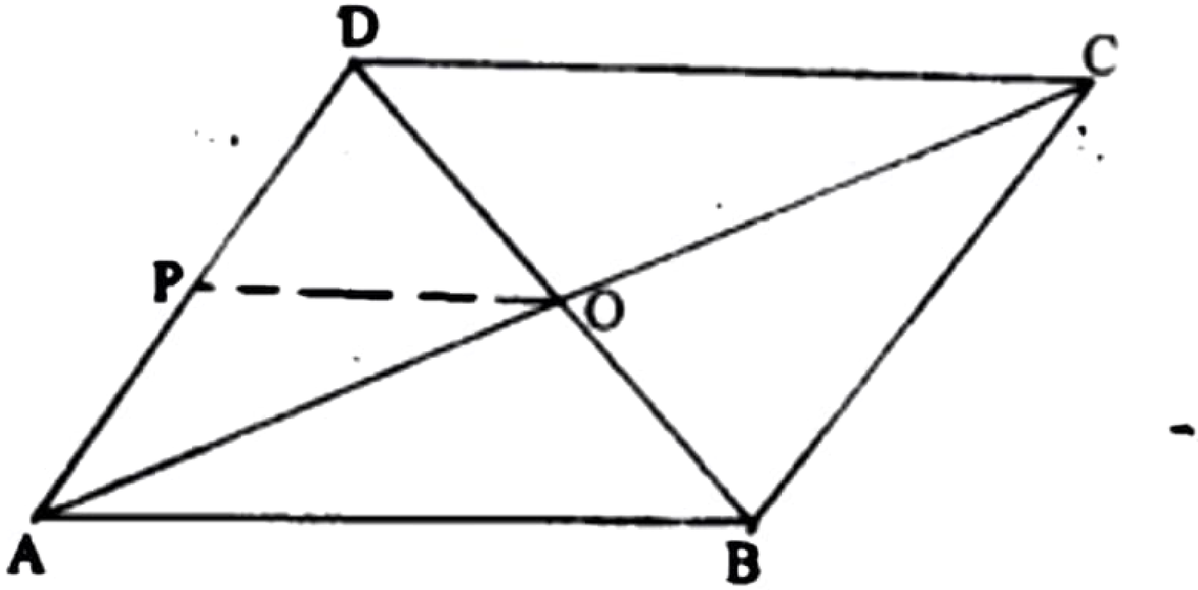
The diagonals AC and BD of a parallelogram ABCD intersect at O. If P is the mid-point of AD, prove that

(i) PQ || AB

(ii) PO= $\frac{1}{2}$ CD.

Solution:

(i) Given : ABCD is a parallelogram in which diagonals AC and BD intersect each other. At point O, P is the mid-point of AD. Join OP.



To Prove : (*i*) PQ || AB (*ii*) PQ = $\frac{1}{2}$ CD.

Proof : We know that in parallelogram diagonals bisect each other.

$\therefore$ BO = OD

i.e. O is the mid-point of BD

Now, in ΔABD,

P and O is the mid-point of AD and BD respectively

$\therefore$ PO || AB and PO = $\frac{1}{2}$ AB ...(1)

i.e. PO || AB [Proved (*i*) part]

(*ii*) Now $\because$ ABCD is a parallelogram

$\therefore$ AB = CD ...(2)

From (1) and (2),

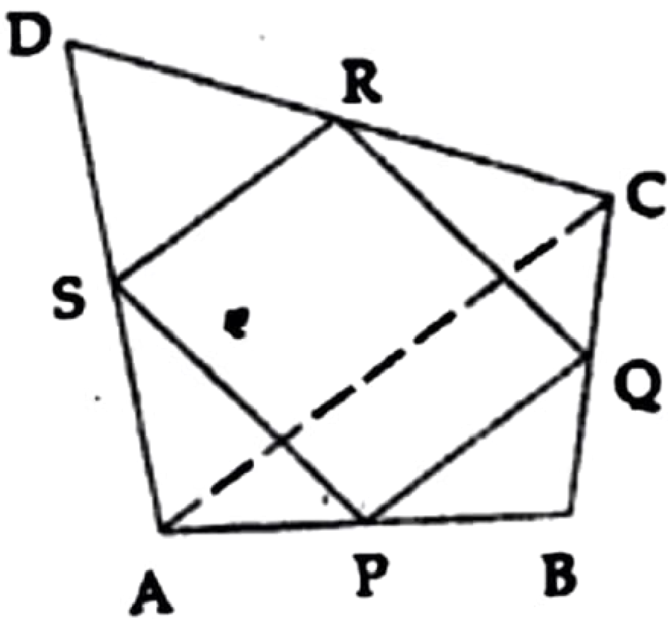
$$PO = \frac{1}{2} CD$$ (Q.E.D.)

In the adjoining figure, ABCD is a quadrilateral in which P, Q, R and S are mid-points of AB, BC, CD and DA respectively. AC is its diagonal. Show that

(i) SR || AC and SR = $\frac{1}{2}$AC

(ii) PQ = SR

(iii) PQRS is a parallelogram.



(*ii*) PQ = SR

(*iii*) PQRS is a parallelogram

Proof : (*i*) In ΔADC

S and R are the mid-points of AD and DC

$\therefore$ $SR \parallel AC$ and $SR = \frac{1}{2} AC$...(*i*)

(Mid-points theorm)

(*ii*) Similarly in ΔABC,

P and Q are mid-points of AB and BC

$PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ ...(*ii*)

From (*i*) and (*ii*),

$PQ = SR$ and $PQ \parallel SR$

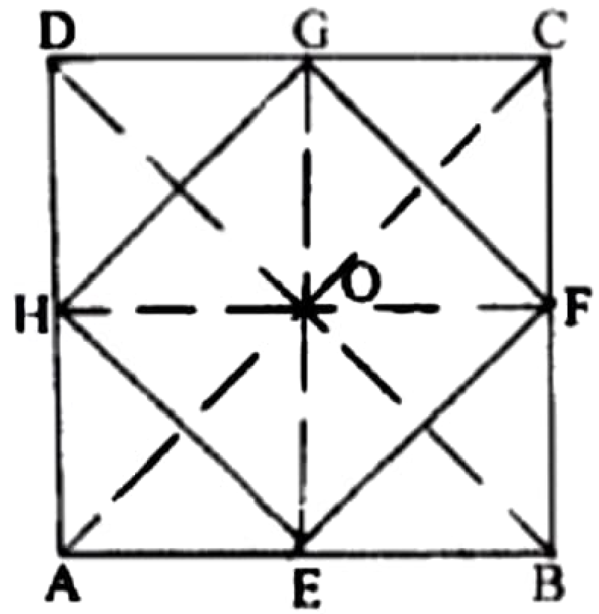
(*iii*) $\because$ $PQ = SR$ and $PQ \parallel SR$

$\therefore$ PQRS is a parallelogram

Show that the quadrilateral formed by joining the mid-points of the adjacent sides of a square, is also a square,

Solution:

Given : A square ABCD in which E, F, G and H are mid-points of AB, BC, CD and DA respectively join EF, FG, GH and HE.



To Prove : EFGH is a square

Construction : Join AC and BD.

Proof : In ΔACD, G and H are mid-points of CD and AC respectively.

$\therefore \quad GH \parallel AC$ and $GH = \frac{1}{2} AC$ ...(1)

Now, in ΔABC, E and F are mid-points of AB and BC respectively.

$\therefore$ EF || AC and $EF = \frac{1}{2}AC$ ...(2)

From (1) and (2),

EF || GH and $EF = GH = \frac{1}{2}AC$ ....(3)

Similarly, we can prove that

EF || GH and $EH = GF = \frac{1}{2}BD$

But AC = BD ($\because$ Diagonals of square are equal)

Dividing both sides by 2,

$\frac{1}{2}AC = \frac{1}{2}BD$ ...(4)

From (3) and (4),

EF = GH = EH = GF ...(5)

$\therefore$ EFGH is a parallelogram

Now, in ΔGOH and ΔGOF

OH = OF

(Diagonals of parallelogram bisect each other)

OG = OG (Common)

GH = GF [From (5)]

$\therefore$ $\Delta GOH \cong \Delta GOF$

[By S.S.S. axiom of congruency]

$\therefore$ $\angle GOH = \angle GOF$ (c.p.c.t.)

Now $\angle GOH + \angle GOF = 180^\circ$ (Linear pair)

or $\angle GOH + \angle GOH = 180^\circ$

$\therefore \quad \Delta GOH \cong \Delta GOF$

[By S.S.S. axiom of congruency]

$\therefore \quad \angle GOH = \angle GOF$ (c.p.c.t.)

Now $\angle GOH + \angle GOF = 180^\circ$ (Linear pair)

or $\angle GOH + \angle GOH = 180^\circ$

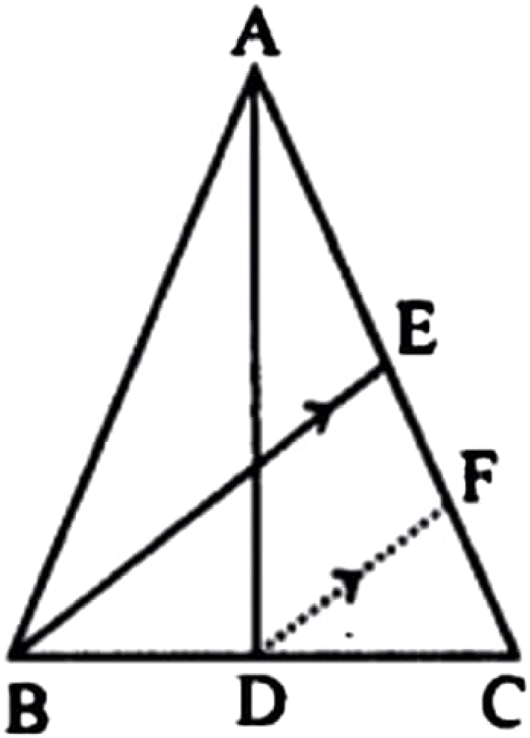
or $2\angle GOH$

$$\therefore \quad \angle GOH = \frac{180^\circ}{2} = 90^\circ$$

$\therefore$ Diagonals of parallelogram ABCD bisect and perpendicular to each other.

$\therefore$ EFGH is a square (Q.E.D.)

In the adjoining figure, AD and BE are medians of ΔABC. If DF U BE, prove that CF = $\frac{1}{4}$ AC.



Solution:

Given : In the given figure,

AD and BE are the medians of ΔABC

DF || BE is drawn

To prove : CF = $\frac{1}{4}$AC

Proof:

In ΔBCE

$\therefore$ D is the mid-point of BC and $DF \parallel BE$

$\therefore$ F is the mid-points of EC

$$\Rightarrow CF = \frac{1}{2} EC \quad ...(i)$$

$\because$ E is the mid-point of AC

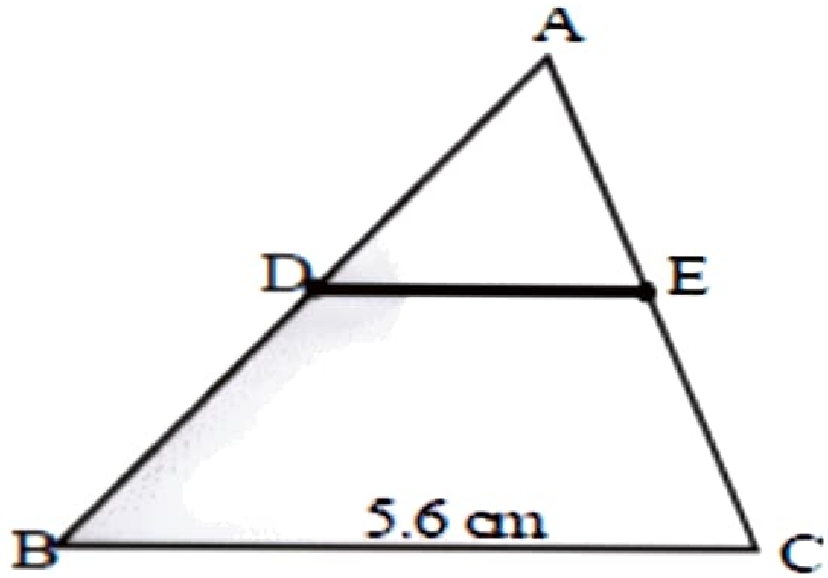
$$\therefore EC = \frac{1}{2} AC \quad ...(ii)$$

From (*i*) and (*ii*),

$$CF = \frac{1}{2} EC = \frac{1}{2}\left(\frac{1}{2} AC\right)$$

$$= \frac{1}{4} AC$$

Ex. In figure, D and E are the mid-point of the sides AB and AC respectively of ΔABC. If BC = 5.6 cm, find DE.



Sol. D is mid-point of AB and E is mid-point of AC.

$$\Rightarrow DE = \frac{1}{2} BC$$

$$= \frac{1}{2} \times 5.6 \text{ cm} = 2.8 \text{ cm}.$$