Question 1: Factorize $x^3 + x - 3x^2 - 3$

Solution:

$x^3 + x - 3x^2 - 3$

Here x is common factor in $x^3 + x$ and $- 3$ is common factor in $- 3x^2 - 3$

$x^3 - 3x^2 + x - 3$

$x^2 (x - 3) + 1(x - 3)$

Taking $(x - 3)$ common

$(x - 3) (x^2 + 1)$

Therefore $x^3 + x - 3x^2 - 3 = (x - 3) (x^2 + 1)$

Question 2: Factorize $a(a + b)^3 - 3a^2b(a + b)$

Solution:

$a(a + b)^3 - 3a^2b(a + b)$

Taking $a (a + b)$ as common factor

$= a(a + b) \{(a + b)^2 - 3ab\}$

$= a(a + b) \{a^2 + b^2 + 2ab - 3ab\}$

$= a(a + b) (a^2 + b^2 - ab)$

Question 3: Factorize $x(x^3 - y^3) + 3xy(x - y)$

Solution:

$x(x^3 - y^3) + 3xy(x - y)$

$= x(x - y) (x^2 + xy + y^2) + 3xy(x - y)$

Taking $x(x - y)$ as a common factor

$= x(x - y) (x^2 + xy + y^2 + 3y)$

$= x(x - y) (x^2 + xy + y^2 + 3y)$

Question 4: Factorize $a^2x^2 + (ax^2 + 1)x + a$

Solution:

$a^2x^2 + (ax^2 + 1)x + a$

$= a^2x^2 + a + (ax^2 + 1)x$

$= a(ax^2 + 1) + x(ax^2 + 1)$

$= (ax^2 + 1) (a + x)$

Question 5: Factorize $x^2 + y - xy - x$

Solution:

$x^2 + y - xy - x$

$= x^2 - x - xy + y$

$= x(x- 1) - y(x - 1)$

$= (x - 1) (x - y)$

Question 6: Factorize $x^3 - 2x^2y + 3xy^2 - 6y^3$

Solution:

$x^3 - 2x^2y + 3xy^2 - 6y^3$

$= x^2(x - 2y) + 3y^2(x - 2y)$

$= (x - 2y) (x^2 + 3y^2)$

Question 7: Factorize $6ab - b^2 + 12ac - 2bc$

Solution:

$6ab - b^2 + 12ac - 2bc$

$= 6ab + 12ac - b^2 - 2bc$

Taking 6a common from first two terms and –b from last two terms

$= 6a(b + 2c) - b(b + 2c)$

Taking (b + 2c) common factor

$= (b + 2c) (6a - b)$

Question 8: Factorize $(x^2 + 1/x^2) - 4(x + 1/x) + 6$

Solution:

$(x^2 + 1/x^2) - 4(x + 1/x) + 6$

$= x^2 + 1/x^2 - 4x - 4/x + 4 + 2$

$= x^2 + 1/x^2 + 4 + 2 - 4/x - 4x$

$= (x^2) + (1/x)^2 + (-2)^2 + 2x(1/x) + 2(1/x)(-2) + 2(-2)x$

As we know, $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x+y+z)^2$

So, we can write;

$= (x + 1/x + (-2))^2$

or $(x + 1/x - 2)^2$

Therefore, $x^2 + 1/x^2) - 4(x + 1/x) + 6 = (x + 1/x - 2)^2$

Question 9: Factorize $x(x - 2)(x - 4) + 4x - 8$

Solution:

$x(x - 2)(x - 4) + 4x - 8$

$= x(x - 2)(x - 4) + 4(x - 2)$

$= (x - 2)[x(x - 4) + 4]$

$= (x - 2)(x^2 - 4x + 4)$

$= (x - 2)[x^2 - 2(x)(2) + (2)^2]$

$= (x - 2)(x - 2)^2$

$= (x - 2)^3$

Question 10: Factorize $(x + 2)(x^2 + 25) - 10x^2 - 20x$

Solution :

$(x + 2)(x^2 + 25) - 10x(x + 2)$

Take $(x + 2)$ as common factor;

$= (x + 2)(x^2 + 25 - 10x)$

$=(x + 2)(x^2 - 10x + 25)$

Expanding the middle term of $(x^2 - 10x + 25)$

$=(x + 2)(x^2 - 5x - 5x + 25)$

$=(x + 2)\{x(x - 5) - 5(x - 5)\}$

$=(x + 2)(x - 5)(x - 5)$

$=(x + 2)(x - 5)^2$

Therefore, $(x + 2)(x^2 + 25) - 10x(x + 2) = (x + 2)(x - 5)^2$

Question 11: Factorize $2a^2 + 2\sqrt{6}\, ab + 3b^2$

Solution:

$2a^2 + 2\sqrt{6}\, ab + 3b^2$

Above expression can be written as $(\sqrt{2}a)^2 + 2 \times \sqrt{2}a \times \sqrt{3}b + (\sqrt{3}b)^2$

As we know, $(p + q)^2 = p^2 + q^2 + 2pq$

Here $p = \sqrt{2}a$ and $q = \sqrt{3}b$

$= (\sqrt{2}a + \sqrt{3}b)^2$

Therefore, $2a^2 + 2\sqrt{6}\, ab + 3b^2 = (\sqrt{2}a + \sqrt{3}b)^2$

Question 12: Factorize $(a - b + c)^2 + (b - c + a)^2 + 2(a - b + c)(b - c + a)$

Solution:

$(a - b + c)^2 + (b - c + a)^2 + 2(a - b + c)(b - c + a)$

{Because $p^2 + q^2 + 2pq = (p + q)^2$}

Here $p = a - b + c$ and $q = b - c + a$

$= [a - b + c + b - c + a]^2$

$= (2a)^2$

$= 4a^2$

Question 13: Factorize $a^2 + b^2 + 2(ab+bc+ca)$

Solution:

$a^2 + b^2 + 2ab + 2bc + 2ca$

As we know, $p^2 + q^2 + 2pq = (p + q)^2$

We get,

$= (a+b)^2 + 2bc + 2ca$

$= (a+b)^2 + 2c(b + a)$

Or $(a+b)^2 + 2c(a + b)$

Take $(a + b)$ as common factor;

$= (a + b)(a + b + 2c)$

Therefore, $a^2 + b^2 + 2ab + 2bc + 2ca = (a + b)(a + b + 2c)$

Question 14: Factorize $4(x-y)^2 - 12(x - y)(x + y) + 9(x + y)^2$

Solution :

Consider $(x - y) = p$, $(x + y) = q$

$= 4p^2 - 12pq + 9q^2$

Expanding the middle term, $-12 = -6 -6$ also $4\times 9=-6 \times -6$

$= 4p^2 - 6pq - 6pq + 9q^2$

$=2p(2p - 3q) -3q(2p - 3q)$

$= (2p - 3q)(2p - 3q)$

$= (2p - 3q)^2$

Substituting back $p = x - y$ and $q = x + y$;

$= [2(x-y) - 3(x+y)]^2 = [2x - 2y - 3x - 3y]^2$

$= (2x-3x-2y-3y)^2$

$=[-x - 5y]^2$

$=[(-1)(x+5y)]^2$

$=(x+5y)^2$

Therefore, $4(x-y)^2 - 12(x - y)(x + y) + 9(x + y)^2 = (x+5y)^2$

Question 15: Factorize $a^2 - b^2 + 2bc - c^2$

Solution :

$a^2 - b^2 + 2bc - c^2$

As we know, $(a-b)^2 = a^2 + b^2 - 2ab$

$= a^2 - (b - c)^2$

Also we know, $a^2 - b^2 = (a+b)(a-b)$

$= (a + b - c)(a - (b - c))$

$= (a + b - c)(a - b + c)$

Therefore, $a^2 - b^2 + 2bc - c^2 =(a + b - c)(a - b + c)$