MATRIX

What is matrix?

In Science, Commerce and in our everyday life also, it is often convenient to represent set of numbers in rows and columns, called arrays. Suppose a factory has three outlets X, Y, Z which supply five products A, B, C, D, E. Suppose X supplies product A, B, C, D, E in number 200, 250, 150, 200 and 250 respectively. The corresponding number produced/supplied by Y and Z are 200, 100, 250, 150, 250 and 250, 300, 150, 100, 100.

This information relating to the supply can be written as—

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccc} X & Y & Z \end{array} \\ \begin{bmatrix} 200 & 200 & 250 \\ 250 & 100 & 300 \\ 150 & 250 & 150 \\ 200 & 150 & 100 \\ 250 & 250 & 100 \end{bmatrix} \end{array}$$

Such an array of numbers arranged in rows and column is called a matrix.

Dimension of Matrix

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots & a_{1m} \\ a_{21} & a_{22} & a_{23} \cdots & a_{2m} \\ \vdots & & & \\ a_{n1} & a_{n2} & a_{n3} \cdots & a_{nm} \end{bmatrix}$$

No of Rows – n

No of Column – m

Dimension → $n \times m$

Use of Matrix in Real life

Matrices are used for plotting graphs, statistics, also to do science problems and scientific studies.

Matrices are also used in representing real world datas like population of people, infant mortality rate etc. They are best representation methods for plotting surveys.

Long Answer Type Questions

1\. i) $A=\begin{pmatrix}2&0&1\\1&0&-1\\0&-1&1\end{pmatrix}$

$$A^2-4A+3I=\begin{pmatrix}2&0&1\\1&0&-1\\0&-1&1\end{pmatrix}\begin{pmatrix}2&0&1\\1&0&-1\\0&-1&1\end{pmatrix}-4\begin{pmatrix}2&0&1\\1&0&-1\\0&-1&1\end{pmatrix}+3\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$$

$$=\begin{pmatrix}4+0+0&0+0-1&2+0+1\\2+0+0&0+0+1&1+0-1\\0-1+0&0+0-1&0+1+1\end{pmatrix}-\begin{pmatrix}8&0&4\\4&0&-4\\0&-4&4\end{pmatrix}+\begin{pmatrix}3&0&0\\0&3&0\\0&0&3\end{pmatrix}$$

$$=\begin{pmatrix}4-8+3&-1+0+0&3-4+0\\2-4+0&1+0+3&0+4+0\\-1+0+0&-1+4+0&2-4+3\end{pmatrix}$$

$$=\begin{pmatrix}-1&-1&-1\\-2&4&4\\-1&3&1\end{pmatrix}$$

ii) $A^2-5A-14I$

$$=\begin{pmatrix}3&-5\\-4&2\end{pmatrix}\begin{pmatrix}3&-5\\-4&2\end{pmatrix}-5\begin{pmatrix}3&-5\\-4&2\end{pmatrix}-14\begin{pmatrix}1&0\\0&1\end{pmatrix}$$

$$=\begin{pmatrix}9+20&-15-10\\-12-8&20+4\end{pmatrix}-\begin{pmatrix}15&-25\\-20&10\end{pmatrix}-\begin{pmatrix}14&0\\0&14\end{pmatrix}$$

$$=\begin{pmatrix}29-15-14&-25+25+0\\-20+20-0&24-10-14\end{pmatrix}$$

$$=\begin{pmatrix}+\square&0\\0&0\end{pmatrix}$$

$$=O$$

2\. $BC=\begin{pmatrix}0&-1&-2\\3&2&-1\\4&-2&0\end{pmatrix}\begin{pmatrix}1&-2&0\\0&1&2\\3&0&-1\end{pmatrix}=\begin{pmatrix}0+0-6&0-1+0&0-2+2\\3+0-3&-6+2+0&0+4+1\\4+0+0&-8-2+0&0-4+0\end{pmatrix}=\begin{pmatrix}-6&-1&0\\0&-4&5\\4&-10&-4\end{pmatrix}$

$$A(BC) = \begin{pmatrix} 2 & 0 & 9 \\ -1 & 6 & 10 \\ 4 & -1 & 2 \end{pmatrix} \begin{pmatrix} -6 & -1 & 0 \\ 0 & -4 & 5 \\ 4 & -10 & -4 \end{pmatrix} = \begin{pmatrix} -12+36 & -2-90 & -36 \\ 6+40 & 1-24-100 & 30-40 \\ -24+8 & -4+4-20 & -5-8 \end{pmatrix} = \begin{pmatrix} 24 & -92 & -36 \\ 46 & -123 & -10 \\ -16 & -20 & -13 \end{pmatrix}$$

$$AB = \begin{pmatrix} 2 & 0 & 9 \\ -1 & 6 & 10 \\ 4 & -1 & 2 \end{pmatrix} \begin{pmatrix} 0 & -1 & -2 \\ 3 & 2 & -1 \\ 4 & -2 & 0 \end{pmatrix} = \begin{pmatrix} 0+0+36 & -2+0-18 & -4+0+0 \\ 0+18+40 & 1+12-20 & 2-6+10 \\ 0-3+8 & -4-2-4 & -8+1+0 \end{pmatrix} = \begin{pmatrix} 36 & -20 & -4 \\ 58 & -7 & \cancel{6}-4 \\ 5 & -10 & -7 \end{pmatrix}$$

$$(AB)C = \begin{pmatrix} 36 & -20 & -4 \\ 58 & -7 & \cancel{6}-4 \\ 5 & -10 & -7 \end{pmatrix} \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 2 \\ 3 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 36-12 & -72-20 & -40+4 \\ 58-12 & -116-7 & -14+4 \\ 5-21 & -10-10 & -20+7 \end{pmatrix} = \begin{pmatrix} 24 & -92 & -36 \\ 46 & -123 & -10 \\ -16 & -20 & -13 \end{pmatrix}$$

clearly, $A(BC) = (AB)C$

3. $AB = BA$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 2 & x & x \\ x & 4 & 5 \\ x & 6 & 7 \end{pmatrix} = \begin{pmatrix} 2 & x & x \\ x & 4 & 5 \\ x & 6 & 7 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2 & x & x \\ -x & -4 & -5 \\ -x & -6 & -7 \end{pmatrix} = \begin{pmatrix} 2 & -x & -x \\ x & -4 & -5 \\ x & -6 & -7 \end{pmatrix}$$

$\Rightarrow x = -x \Rightarrow 2x = 0 \Rightarrow x = 0.$

4.

5. $A+I_3 = \begin{pmatrix} 1 & 3 & 4 \\ -1 & 1 & 3 \\ -2 & -3 & 1 \end{pmatrix}$

$A = \begin{pmatrix} 1 & 3 & 4 \\ -1 & 1 & 3 \\ -2 & -3 & 1 \end{pmatrix} - I_3 = \begin{pmatrix} 1 & 3 & 4 \\ -1 & 1 & 3 \\ -2 & -3 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 3 & 4 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{pmatrix}$

$A - I_3 = \begin{pmatrix} 0 & 3 & 4 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 3 & 4 \\ -1 & -1 & 3 \\ -2 & -3 & -1 \end{pmatrix}$

$(A+I_3)\cdot(A-I_3)$

$= \begin{pmatrix} 1 & 3 & 4 \\ -1 & 1 & 3 \\ -2 & -3 & 1 \end{pmatrix} \begin{pmatrix} -1 & 3 & 4 \\ -1 & -1 & 3 \\ -2 & -3 & -1 \end{pmatrix} = \begin{pmatrix} -1-3-8 & 3-3-12 & 4+9-4 \\ 1-1-6 & -3-1-9 & -4+3-3 \\ 2+3-2 & -6+3-3 & -8-9-1 \end{pmatrix} = \begin{pmatrix} -12 & -12 & 9 \\ -6 & -13 & -4 \\ 3 & -6 & -18 \end{pmatrix}$

6. i) $f(x) = 2x^2+3x+5$

$f(A) = 2A^2+3A+5 = 2\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} + 3\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} + 5\cdot I$

$= 2\begin{pmatrix} 4+3 & 2+4 \\ 6+12 & 3+16 \end{pmatrix} + \begin{pmatrix} 6 & 3 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$

$= \begin{pmatrix} 14 & 12 \\ 36 & 38 \end{pmatrix} + \begin{pmatrix} 11 & 3 \\ 9 & 17 \end{pmatrix}$

$= \begin{pmatrix} 25 & 15 \\ 45 & 55 \end{pmatrix}$

(ii) $f(A) = A^2 - 2A - 3I$.

$= \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} - 2\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} - 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1+4 & 2+2 \\ 2+2 & 4+1 \end{pmatrix} - \begin{pmatrix} 2 & 4 \\ 4 & 2 \end{pmatrix} - \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

$= \begin{pmatrix} 5-2-3 & 4-4-0 \\ 4-4-0 & 5-2-3 \end{pmatrix}$

$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

7. $A^2 + 2I_3 = 3A$

$$\Rightarrow \begin{pmatrix} 1 & x & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & x & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{pmatrix} = 3\begin{pmatrix} 1 & x & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{pmatrix} - 2\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1+2x+0 & x+2x+0 & -2+4x-4 \\ 2+4+0 & 2x+4+0 & -4+8+8 \\ 0+0+0 & 0+0+0 & 0+0+4 \end{pmatrix} = \begin{pmatrix} 3-2 & 3x & -6 \\ 6 & 6-2 & 12 \\ 0 & 0 & 6-2 \end{pmatrix}$$

$\Rightarrow 1+2x = 3-2$

$\Rightarrow 2x = 0$

$\Rightarrow x = 0$

8. i) $AB = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 2-1 & -4+1 \\ 3-4 & -6+4 \end{pmatrix} = \begin{pmatrix} 1 & -3 \\ -1 & -2 \end{pmatrix}$ $\therefore (AB)^T = \begin{pmatrix} 1 & -1 \\ -3 & -2 \end{pmatrix}$

$A^T = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ $B^T = \begin{pmatrix} 1 & -1 \\ -2 & 1 \end{pmatrix}$

$B^T A^T = \begin{pmatrix} 1 & -1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 2-1 & 3-4 \\ -4+1 & -6+4 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -3 & -2 \end{pmatrix}$

$\therefore (AB)^T = B^T A^T$

(ii) $AB = \begin{pmatrix} -2 & 1 & 3 \\ 0 & 4 & -1 \end{pmatrix}_{2\times 3}\begin{pmatrix} 2 & 1 \\ -3 & 0 \\ 4 & -5 \end{pmatrix}_{3\times 2} = \begin{pmatrix} -4-3+12 & -2+0-15 \\ 0-12-4 & 0+0+5 \end{pmatrix} = \begin{pmatrix} 5 & -17 \\ -16 & 5 \end{pmatrix}$

$(AB)' = \begin{pmatrix} 5 & -16 \\ -17 & 5 \end{pmatrix}$

$B'A' = \begin{pmatrix} 2 & -3 & 4 \\ 1 & 0 & -5 \end{pmatrix}\begin{pmatrix} -2 & 0 \\ 1 & 4 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} -4-3+12 & 0-12-4 \\ -2+0-15 & 0+0+5 \end{pmatrix} = \begin{pmatrix} 5 & -16 \\ -17 & 5 \end{pmatrix}$

(iii) $AB = \begin{pmatrix} 1 & 2 & 5 \\ -1 & 3 & -4 \end{pmatrix}_{2\times 3}\begin{pmatrix} 3 & -2 & 1 \\ 0 & -1 & 4 \\ 5 & 2 & -1 \end{pmatrix}_{3\times 2} = \begin{pmatrix} 3+0+25 & -2-2+10 & 1+8-5 \\ -3+0-20 & 2-3-8 & -1+12+4 \end{pmatrix} = \begin{pmatrix} 28 & 6 & 4 \\ -23 & -9 & 15 \end{pmatrix}$

$(AB)' = \begin{pmatrix} 28 & -23 \\ 6 & -9 \\ 4 & 15 \end{pmatrix}$

$$B^TA^T = \begin{pmatrix} 3 & 0 & 5 \\ -2 & -1 & 2 \\ 1 & 4 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 3 \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} 3+0+25 & -3+0-20 \\ -2-2+10 & 2-3-8 \\ 1+8-5 & -1+12+4 \end{pmatrix} = \begin{pmatrix} 28 & -23 \\ 6 & -9 \\ 4 & 15 \end{pmatrix}$$

$$(AB)^T = B^TA^T$$

iv) $$AB = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}_{3\times1} \begin{pmatrix} -2 & -1 & -4 \end{pmatrix}_{1\times3} = \begin{pmatrix} 2 & 1 & 4 \\ -4 & -2 & -8 \\ -6 & -3 & -12 \end{pmatrix}$$

$$(AB)^T = \begin{pmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{pmatrix}$$

$$B^TA^T = \begin{pmatrix} -2 \\ -1 \\ -4 \end{pmatrix} \begin{pmatrix} -1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{pmatrix} \qquad \therefore (AB)^T = B^TA^T$$

9. $$A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$$

$$AA' = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} = \begin{pmatrix} \cos^2\alpha + \sin^2\alpha & -\sin\alpha\cos\alpha + \sin\alpha\cos\alpha \\ -\sin\alpha\cos\alpha + \sin\alpha\cos\alpha & \sin^2\alpha + \cos^2\alpha \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I.$$

A^{-1} is said to be inverse of A if $A \cdot A^{-1} = I$.

here $A^{-1} = A' = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$

10. i) $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ $\quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$A^2 - 5A + 7I$

$$= \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} - 5\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} + 7\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{pmatrix} - \begin{pmatrix} 15 & 5 \\ -5 & 10 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0 \quad \text{(Proved).}$$

$A^2 - 5A + 7I = 0$

$\Rightarrow A^2 \cdot A^{-1} - 5A \cdot A^{-1} + 7IA^{-1} = 0 \cdot A^{-1}$

$\Rightarrow A - 5I + 7A^{-1} = 0$

$\Rightarrow 7A^{-1} = 5I - A$

$\Rightarrow A^{-1} = \frac{1}{7}(5I - A)$

So, $A^{-1} = \frac{1}{7}\left\{5\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - \begin{pmatrix}3 & 1\\ -1 & 2\end{pmatrix}\right\} = \frac{1}{7}\begin{pmatrix}5-3 & 0-1\\ 0+1 & 5-2\end{pmatrix} = \begin{pmatrix}2/7 & -1/7\\ 1/7 & 3/7\end{pmatrix}$

ii) $A^2 = \begin{pmatrix}4 & 5\\ 5 & 6\end{pmatrix}\begin{pmatrix}4 & 5\\ 5 & 6\end{pmatrix} = \begin{pmatrix}16+25 & 20+30\\ 20+30 & 25+36\end{pmatrix} = \begin{pmatrix}41 & 50\\ 50 & 61\end{pmatrix}$

$10A + I = 10\begin{pmatrix}4 & 5\\ 5 & 6\end{pmatrix} + \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = \begin{pmatrix}41 & 50\\ 50 & 61\end{pmatrix}$

So, $A^2 = 10A + I$.

$\Rightarrow 10A + I = A^2$

$\Rightarrow (10A + I)A^{-1} = A^2 \cdot A^{-1} \Rightarrow 10 \cdot A \cdot A^{-1} + I \cdot A^{-1} = A \cdot AA^{-1} \Rightarrow 10I + A^{-1} = AI$

$\Rightarrow A^{-1} = A - 10I$

$= \begin{pmatrix}4 & 5\\ 5 & 6\end{pmatrix} - 10\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$= \begin{pmatrix}4-10 & 5-0\\ 5-0 & 6-10\end{pmatrix} = \begin{pmatrix}-6 & 5\\ 5 & -4\end{pmatrix}$

(iii) $A^2 - 6A + 17I = 0$

$= \begin{pmatrix}2 & -3\\ 3 & 4\end{pmatrix}\begin{pmatrix}2 & -3\\ 3 & 4\end{pmatrix} - 6\begin{pmatrix}2 & -3\\ 3 & 4\end{pmatrix} + 17\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$= \begin{pmatrix}4-9 & -6-12\\ 6+12 & -9+16\end{pmatrix} - \begin{pmatrix}12 & -18\\ 18 & 24\end{pmatrix} + \begin{pmatrix}17 & 0\\ 0 & 17\end{pmatrix}$

$= \begin{pmatrix}-5-12+17 & -18+18+0\\ 18-18+0 & 7-24+17\end{pmatrix}$

$= \begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$

$= 0$

$A^2 - 6A + 17I = 0$

$\Rightarrow A^{-1}(A^2 - 6A + 17I) = A^{-1} \cdot 0$

$\Rightarrow A - 6I + 17A^{-1} = 0$

$\Rightarrow 17A^{-1} = 6I - A = 6\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - \begin{pmatrix}2 & -3\\ 3 & 4\end{pmatrix} = \begin{pmatrix}4 & 3\\ -3 & 2\end{pmatrix}$

$\Rightarrow A^{-1} = \frac{1}{17}\begin{pmatrix}4 & 3\\ -3 & 2\end{pmatrix}$

11. $AA' = \begin{pmatrix} \frac{a}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{b}{3} \\ \frac{2}{3} & \frac{c}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} \frac{a}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{c}{3} \\ \frac{2}{3} & \frac{b}{3} & \frac{1}{3} \end{pmatrix} = I$

$$\Rightarrow \begin{pmatrix} \frac{a^2}{9}+\frac{4}{9}+\frac{4}{9} & \frac{2a}{9}+\frac{2}{9}+\frac{2b}{9} & \frac{2a}{9}+\frac{2c}{9}+\frac{2}{9} \\ \frac{2a}{9}+\frac{2}{9}+\frac{2b}{9} & \frac{4}{9}+\frac{1}{9}+\frac{b^2}{9} & \frac{4}{9}+\frac{c}{9}+\frac{b}{9} \\ \frac{2a}{9}+\frac{2c}{9}+\frac{2}{9} & \frac{4}{9}+\frac{c}{9}+\frac{b}{9} & \frac{4}{9}+\frac{c^2}{9}+\frac{1}{9} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\therefore \frac{a^2}{9}+\frac{4}{9}+\frac{4}{9} = 1$

$\Rightarrow \frac{a^2}{9} = 1-\frac{8}{9} \Rightarrow \frac{a^2}{\cancel{9}} = \frac{1}{\cancel{9}} \Rightarrow a = \pm 1$

now, $\frac{4}{9}+\frac{1}{9}+\frac{b^2}{9} = 1$

$\Rightarrow \frac{b^2}{\cancel{9}} = 1-\frac{5}{9} = \frac{4}{\cancel{9}} \Rightarrow b = \pm 2$

and $\frac{4}{9}+\frac{c^2}{9}+\frac{1}{9} = 1$

$\Rightarrow \frac{c^2}{\cancel{9}} = 1-\frac{5}{9} = \frac{4}{\cancel{9}} \Rightarrow c = \pm 2$

12. $A = \begin{pmatrix} 1 & -2 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 3 & -1 \\ 0 & -1 & -2 \end{pmatrix}$

$$A'B = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 2 & 3 & -1 \\ 0 & -1 & -2 \end{pmatrix} = \begin{pmatrix} 1+0+0 & 2+0+0 & 0+0+0 \\ -2+2+0 & -4+3+0 & 0-1+20 \\ 2-2+0 & 4-3-1 & 0+1-2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & -1 \end{pmatrix}$$

$$(A'B)A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & -2 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0+0 & -2+2+0 & 2-2+0 \\ 0+0+0 & 0-1+0 & 0+1-1 \\ 0+0+0 & 0+0+0 & 0+0-1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$ a diagonal matrix.

13. $A = \begin{pmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{pmatrix}$ $A^T = \begin{pmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ -1 & 7 & 1 \end{pmatrix}$

$A + A^T = \begin{pmatrix} 8 & 5 & 0 \\ 5 & 10 & 5 \\ 0 & 5 & 2 \end{pmatrix}$ $\frac{1}{2}(A+A^T) = \begin{pmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{pmatrix}$ a symmetric matrix

$A - A^T = \begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & 9 \\ 2 & -9 & 0 \end{pmatrix}$ $\frac{1}{2}(A-A^T) = \begin{pmatrix} 0 & -\frac{1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & -\frac{9}{2} & 0 \end{pmatrix}$ a skew symmetric matrix.

$A = \frac{1}{2}(A+A^T) + \frac{1}{2}(A-A^T)$

$\Rightarrow \begin{pmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{pmatrix} + \begin{pmatrix} 0 & -\frac{1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & -\frac{9}{2} & 0 \end{pmatrix}$

14i) $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ ie $A^1 = \begin{pmatrix} 1 & 2\cdot 1 \\ 0 & 1 \end{pmatrix}$ true for $n=1$

$A^2 = A \cdot A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1+0 & 2+2 \\ 0+0 & 0+1 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2\cdot 2 \\ 0 & 1 \end{pmatrix}$ true for $n=2$

let $A^n = \begin{pmatrix} 1 & 2n \\ 0 & 1 \end{pmatrix}$ is true for $n=m$

then $A^m = \begin{pmatrix} 1 & 2m \\ 0 & 1 \end{pmatrix}$

now $A^{m+1} = A^m \cdot A = \begin{pmatrix} 1 & 2m \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1+0 & 2+2m \\ 0+0 & 0+1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2(m+1) \\ 0 & 1 \end{pmatrix}$

so, it is for $n=m+1$

$A^n = \begin{pmatrix} 1 & 2n \\ 0 & 1 \end{pmatrix}$ is true for $n=1,2$

and it is true for $n=m+1$ when it is true for $n=m$

so it is true for all $n \in \mathbb{N}$.

(ii) $A^n = \begin{pmatrix} 1+2n & -4n \\ n & 1-2n \end{pmatrix}$

$n=1$, $A^1 = \cancel{\begin{pmatrix} 1+2\cdot1 & -4\cdot1 \\ 1 & 1-2\cdot1 \end{pmatrix}} = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1+2\cdot1 & -4\cdot1 \\ 1 & 1-2\cdot1 \end{pmatrix}$

so it is true for $n=1$

now $n=2$, $A^2 = A\cdot A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 9-4 & -12+4 \\ 3-1 & -4+1 \end{pmatrix} = \begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}$

$= \begin{pmatrix} 1+2\cdot2 & -4\cdot2 \\ 2 & 1-2\cdot2 \end{pmatrix}$

so it is also true for $n=2$

let it is true for $n=m$

then $A^m = \begin{pmatrix} 1+2m & -4m \\ m & 1-2m \end{pmatrix}$

now, $A^{m+1} = A^m\cdot A = \begin{pmatrix} 1+2m & -4m \\ m & 1-2m \end{pmatrix}\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$

$= \begin{pmatrix} 3+6m-4m & -4-8m+4m \\ 3m+1-2m & -4m-1+2m \end{pmatrix}$

$= \begin{pmatrix} 3+2m & -4-4m \\ m+1 & -2m-1 \end{pmatrix}$

$= \begin{pmatrix} 1+2(m+1) & -4(m+1) \\ (m+1) & 1-2(m+1) \end{pmatrix}$

so it is true for $n=m+1$

given eq^n is true for $n=1,2$ and it is true for $n=m+1$ when it is true for $n=m$

so it is true for all $n \in \mathbb{N}$

15. $A = \begin{pmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{pmatrix}$ clearly its true for $n=1$ $\quad A^n = \begin{pmatrix} \cos n\theta & i\sin n\theta \\ i\sin n\theta & \cos n\theta \end{pmatrix}$

$A^2 = A\cdot A = \begin{pmatrix} \cos\theta & i\sin \cancel{n}\theta \\ i\sin\cancel{n}\theta & \cos\cancel{n}\theta \end{pmatrix}\begin{pmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{pmatrix}$

$= \begin{pmatrix} \cos^2\theta + i^2\sin^2\theta & i(\sin\theta\cos\theta + \sin\theta\cos\theta) \\ i(\sin\theta\cos\theta + \cos\theta\sin\theta) & i^2\sin^2\theta + \cos^2\theta \end{pmatrix} = \begin{pmatrix} \cos^2\theta - \sin^2\theta & 2i\sin\theta\cos\theta \\ 2i\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{pmatrix}$

$$= \begin{pmatrix} \cos 2\theta & i\sin 2\theta \\ i\sin 2\theta & \cos 2\theta \end{pmatrix}$$ so it is true for $n=2$

let, it is true for $n=m$

then $$A^m = \begin{pmatrix} \cos m\theta & i\sin m\theta \\ i\sin m\theta & \cos m\theta \end{pmatrix}$$

now A^{m+1}

$$= A^m \cdot A = \begin{pmatrix} \cos m\theta & i\sin m\theta \\ i\sin m\theta & \cos m\theta \end{pmatrix} \begin{pmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos m\theta \cos\theta + i^2 \sin m\theta \sin\theta & i(\cos m\theta \sin\theta + \sin m\theta \cos\theta) \\ i(\sin m\theta \cos\theta + \cos\theta \sin m\theta) & i^2 \sin m\theta \sin\theta + \cos m\theta \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos m\theta \cos\theta - \sin m\theta \sin\theta & i\sin(m+1)\theta \\ i(\sin(m+1)\theta) & \cos m\theta \cos\theta - \sin m\theta \sin\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos(m+1)\theta & i\sin(m+1)\theta \\ i\sin(m+1)\theta & \cos(m+1)\theta \end{pmatrix}$$

so clearly it is true for $n=m+1$

given relation is true for $n=1,2$ and when it is true for $n=m$ then it is true for $n=m+1$
so it is true for all $n \in \mathbb{N}$.

16. Given relation $$A^n = \begin{bmatrix} a^n & \frac{b(a^n-1)}{a-1} \\ 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a^1 & \frac{b(a^1-1)}{a-1} \\ 0 & 1 \end{bmatrix}$$ so it is true for $n=1$

$$A^2 = A \cdot A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a^2 & ab+b \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a^2 & b(a+1) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a^2 & \frac{b(a+1)(a-1)}{a-1} \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} a^2 & \frac{b(a^2-1)}{a-1} \\ 0 & 1 \end{bmatrix}$$

it is true for $n=2$

let it is true for $n=m$

$$\therefore A^m = \begin{bmatrix} a^m & \frac{b(a^m-1)}{a-1} \\ 0 & 1 \end{bmatrix}$$

$$A^{m+1} = A^m . A = \begin{bmatrix} a^m & \frac{b(a^m-1)}{a-1} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} a^m . a + 0 & a^m . b + \frac{b(a^m-1)}{a-1} \\ 0+0 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} a^{m+1} & b\left\{ a^m + \frac{a^{m-1}}{a-1} \right\} \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} a^{m+1} & b \, \frac{a^m a - \cancel{a^m} + \cancel{a^m} - \cancel{1}1}{a-1} \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} a^{m+1} & \frac{b(a^{m+1}-1)}{a-1} \\ 0 & 1 \end{bmatrix}$$

clearly it is true for $n = m+1$

so given relation it true for $n = 1, 2$ and it is true for $n = m+1$ while true for $n = m$.

so it is true for all $n \in \mathbb{N}$.

17. $$A^n = \begin{pmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{pmatrix}$$

$$A^1 = \begin{pmatrix} 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \end{pmatrix} = \begin{pmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

it is true for $n = 1$

$$A^2 = A . A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1+1+1 & 1+1+1 & 1+1+1 \\ 1+1+1 & 1+1+1 & 1+1+1 \\ 1+1+1 & 1+1+1 & 1+1+1 \end{pmatrix} = \begin{pmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{pmatrix}$$ 18

$$= \begin{pmatrix} 3^{2-1} & 3^{2-1} & 3^{2-1} \\ 3^{2-1} & 3^{2-1} & 3^{2-1} \\ 3^{2-1} & 3^{2-1} & 3^{2-1} \end{pmatrix}$$

$\therefore$ it is true for $n = 2$

let given relation is true for $n=m$

$$A^m = \begin{pmatrix} 3^{m-1} & 3^{m-1} & 3^{m-1} \\ 3^{m-1} & 3^{m-1} & 3^{m-1} \\ 3^{m-1} & 3^{m-1} & 3^{m-1} \end{pmatrix}$$

$$A^{m+1} = A^m \cdot A = \begin{pmatrix} 3^{m-1} & 3^{m-1} & 3^{m-1} \\ 3^{m-1} & 3^{m-1} & 3^{m-1} \\ 3^{m-1} & 3^{m-1} & 3^{m-1} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3^{m-1}+3^{m-1}+3^{m-1} & 3^{m-1}+3^{m-1}+3^{m-1} & 3^{m-1}+3^{m-1}+3^{m-1} \\ 3^{m-1}+3^{m-1}+3^{m-1} & 3^{m-1}+3^{m-1}+3^{m-1} & 3^{m-1}+3^{m-1}+3^{m-1} \\ 3^{m-1}+3^{m-1}+3^{m-1} & 3^{m-1}+3^{m-1}+3^{m-1} & 3^{m-1}+3^{m-1}+3^{m-1} \end{pmatrix}$$

$$= \begin{pmatrix} 3\cdot 3^{m-1} & 3\cdot 3^{m-1} & 3\cdot 3^{m-1} \\ 3\cdot 3^{m-1} & 3\cdot 3^{m-1} & 3\cdot 3^{m-1} \\ 3\cdot 3^{m-1} & 3\cdot 3^{m-1} & 3\cdot 3^{m-1} \end{pmatrix}$$

$$= \begin{pmatrix} 3^m & 3^m & 3^m \\ 3^m & 3^m & 3^m \\ 3^m & 3^m & 3^m \end{pmatrix}$$

$$= \begin{pmatrix} 3^{(m+1)-1} & 3^{(m+1)-1} & 3^{(m+1)-1} \\ 3^{(m+1)-1} & 3^{(m+1)-1} & 3^{(m+1)-1} \\ 3^{(m+1)-1} & 3^{(m+1)-1} & 3^{(m+1)-1} \end{pmatrix}$$

Given relation is true for $n=m+1$ while it is true for $n=m$ and it is also true for $n=1,2$.

so it is true for all $n \in \mathbb{N}$

18. $$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$$

$$A^2 - 4A - 5I_3 = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix} - 4\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix} - 5\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{pmatrix} - \begin{pmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{pmatrix} - \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 9-4-5 & 8-8-0 & 8-8-0 \\ 8-8-0 & 9-4-5 & 8-8-0 \\ 8-8-0 & 8-8-0 & 9-4-5 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= O$$

now $A^2 - 4A - 5I_3 = O$

$\Rightarrow (A^2 - 4A - 5I_3)A^{-1} = O \cdot A^{-1}$

$\Rightarrow A \cdot A \cdot A^{-1} - 4A \cdot A^{-1} - 5I_3 A^{-1} = O$

$\Rightarrow AI - 4I - 5A^{-1} = O$

$\Rightarrow A - 4I = 5A^{-1}$

$\Rightarrow 5A^{-1} = A - 4I$

$A - 4I$

$$= \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix} - 4\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{5}\begin{pmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{pmatrix}$$

19. $A = \begin{pmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{pmatrix}$ $\quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$I + A = \begin{pmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{pmatrix}$$

$$I - A = \begin{pmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{pmatrix}$$

$$(I-A)\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}=\begin{pmatrix}1 & \tan\frac{\alpha}{2}\\ -\tan\frac{\alpha}{2} & 1\end{pmatrix}\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}$$

$$=\begin{pmatrix}\cos\alpha+\sin\alpha\tan\frac{\alpha}{2} & -\sin\alpha+\cos\alpha\tan\frac{\alpha}{2}\\ -\cos\alpha\tan\frac{\alpha}{2}+\sin\alpha & \sin\alpha\tan\frac{\alpha}{2}+\cos\alpha\end{pmatrix}$$

$$=\begin{pmatrix}2\cos^2\frac{\alpha}{2}-1+2\sin\frac{\alpha}{2}\cancel{\cos\frac{\alpha}{2}}\dfrac{\sin\frac{\alpha}{2}}{\cancel{\cos\frac{\alpha}{2}}} & -\sin\alpha+\dfrac{\cos\alpha\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\\ -\dfrac{\cos\alpha\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}+\sin\alpha & 2\sin\frac{\alpha}{2}\cancel{\cos\frac{\alpha}{2}}\dfrac{\sin\frac{\alpha}{2}}{\cancel{\cos\frac{\alpha}{2}}}+2\cos^2\frac{\alpha}{2}-1\end{pmatrix}$$

$$=\begin{pmatrix}2\left\{\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}\right\}-1 & \dfrac{-\sin\alpha\cos\frac{\alpha}{2}+\cos\alpha\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\\ \dfrac{-\cos\alpha\sin\frac{\alpha}{2}+\sin\alpha\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & 2\left(\sin^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}\right)-1\end{pmatrix}$$

$$=\begin{pmatrix}2\cdot1-1 & -\dfrac{\sin(\alpha-\alpha/2)}{\cos\alpha/2}\\ \dfrac{\sin(\alpha-\alpha/2)}{\cos\alpha/2} & 2\cdot1-1\end{pmatrix}$$

$$=\begin{pmatrix}1 & -\dfrac{\sin\alpha/2}{\cos\alpha/2}\\ \dfrac{\sin\alpha/2}{\cos\alpha/2} & 1\end{pmatrix}$$

$$=\begin{pmatrix}1 & -\tan\alpha/2\\ \tan\alpha/2 & 1\end{pmatrix}=I+A \quad \text{(proved)}$$

20. $2I+3E$

$$=2\begin{pmatrix}1&0\\0&1\end{pmatrix}+3\begin{pmatrix}0&1\\0&0\end{pmatrix}=\begin{pmatrix}2&3\\0&2\end{pmatrix}$$

$$(2I+3E)^2=(2I+3E)(2I+3E)=\begin{pmatrix}2&3\\0&2\end{pmatrix}\begin{pmatrix}2&3\\0&2\end{pmatrix}=\begin{pmatrix}4+0&6+6\\0+0&0+4\end{pmatrix}=\begin{pmatrix}4&12\\0&4\end{pmatrix}$$

$$(2I+3E)^3=(2I+3E)^2\cdot(2I+3E)=\begin{pmatrix}4&12\\0&4\end{pmatrix}\begin{pmatrix}2&3\\0&2\end{pmatrix}=\begin{pmatrix}8+0&12+24\\0+0&0+8\end{pmatrix}=\begin{pmatrix}8&36\\0&8\end{pmatrix}$$

$8I + 36E$

$$= 8\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + 36\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & 36 \\ 0 & 8 \end{pmatrix}$$

$$\therefore \; (2I + 3E)^3 = 8I + 36E$$