Quadratic Equation Solving Process

4. i) $3y^2 - 20 = 160 - 2y^2$

$\Rightarrow 5y^2 = 180$

$\Rightarrow y^2 = 36$

$\Rightarrow y = \pm 6$

ii) $(2x+1)^2 + (x+1)^2 = 6x + 47$

$\Rightarrow 4x^2 + 4x + 1 + x^2 + 2x + 1 = 6x + 47$

$\Rightarrow 5x^2 = 45$

$\Rightarrow x^2 = 9$

$\Rightarrow x = \pm 3$

iii) $(x-7)(x-9) = 195$

$\Rightarrow x^2 - 16x + 63 = 195$

$\Rightarrow x^2 - 16x - 132 = 0$

$\Rightarrow x^2 - 22x + 6x - 132 = 0$

$\Rightarrow (x-22)(x+6) = 0$

$\Rightarrow x = 22$ or $x = -6$

iv) $3x - \frac{3\times 8}{x} = \frac{x}{3}$, $x \neq 0$

$\Rightarrow (3x^2 - 24)\,3 = x^2$

$\Rightarrow 8x^2 = 72$

$\Rightarrow x^2 = 9 \Rightarrow x = \pm 3$

v) $\frac{x}{3} + \frac{3}{x} = \frac{15}{x}$, $x \neq 0$

$\Rightarrow \frac{x}{3} = \frac{12}{x}$

$\Rightarrow x^2 = 36$

$\Rightarrow x = \pm 6$

vi) $10x - \frac{1}{x} = 3$, $x \neq 0$

$\Rightarrow 10x^2 - 3x - 1 = 0$

$\Rightarrow 10x^2 - 5x + 2x - 1 = 0$

$\Rightarrow 5x(2x-1) + 1(2x-1) = 0$

$\Rightarrow (2x-1)(5x+1) = 0$ $\Rightarrow$ either, $2x-1=0$ ie $x = 1/2$

or, $5x+1=0$ ie $x = -1/5$

vii) $\frac{2}{x^2} - \frac{5}{x} + 2 = 0$, $x \neq 0$

$\Rightarrow 2 - 5x + 2x^2 = 0$

$\Rightarrow 2x^2 - 4x - x + 2 = 0$

$\Rightarrow 2x(x-2) - 1(x-2) = 0$

$\Rightarrow (x-2)(2x-1) = 0$

$\Rightarrow$ either $x-2=0$ or $2x-1=0$

$\Rightarrow$ either $x=2$ or $x = 1/2$.

viii) $\left(\frac{x-2}{x+2}\right) + 6\left(\frac{x-2}{x-6}\right) = 1$, $x \neq -2, 6$

$\Rightarrow (x-2)\left\{\frac{1}{x+2} + \frac{6}{x-6}\right\} = 1$

$\Rightarrow (x-2)\left(\frac{x-6+12+6x}{x^2-4x-12}\right) = 1$

$\Rightarrow (x-2)(7x+6) = x^2 - 4x - 12$

$\Rightarrow 7x^2 - 8x - 12 = x^2 - 4x - 12$

$\Rightarrow 6x^2 - 4x = 0 \Rightarrow 2x(3x-2) = 0 \Rightarrow x(3x-2) = 0 \Rightarrow$ either $x=0$ or $x = 2/3$

Solve:

ix) $\frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{6}$, $x \neq 3, -5$

$\Rightarrow \frac{x+5-x+3}{x^2+2x-15} = \frac{1}{6}$

$\Rightarrow x^2+2x-15 = 48$

$\Rightarrow x^2+2x-63 = 0$

$\Rightarrow x^2+9x-7x-63 = 0$

$\Rightarrow (x+9)(x-7) = 0$

$\Rightarrow$ either $x = -9$ or $x = 7$.

Solve:

x) $\frac{x}{x+1} + \frac{x+1}{x} = 2\frac{1}{12}$, $x \neq 0, -1$

$\Rightarrow \frac{x^2+x^2+2x+1}{x^2+x} = \frac{25}{12}$

$\Rightarrow \frac{2x^2+2x+1}{x^2+x} = \frac{25}{12}$

$\Rightarrow 25x^2+25x = 24x^2+24x+12$

$\Rightarrow x^2+x-12 = 0$

$\Rightarrow (x+4)(x-3) = 0$

$\Rightarrow$ either $x = -4$ or $x = 3$.

Solve:

xi) $\frac{ax+b}{a+bx} = \frac{cx+d}{c+dx}$ $(a \neq b, c \neq d), (x \neq -\frac{a}{b}, -\frac{c}{d})$

$\Rightarrow acx+adx^2+bc+bdx = acx+ad+bcx^2+bdx$

$\Rightarrow (ad-bc)x^2 = (ad-bc)$

$\Rightarrow x^2 = \frac{(ad-bc)}{(ad-bc)}$ since, $a \neq b$, $c \neq d$

$\Rightarrow x^2 = 1$

$\Rightarrow x = \pm 1$

xii) $(2x+1) + \frac{3}{(2x+1)} = 4$

$\Rightarrow a + \frac{3}{a} = 4$ (let $2x+1 = a$)

$\Rightarrow a^2-4a+3 = 0$

$\Rightarrow (a-3)(a-1) = 0 \Rightarrow$ either $a = 3$ or $a = 1$

when $a=3 \Rightarrow 2x+1=3 \Rightarrow x=1$

when $a=1 \Rightarrow 2x+1=1 \Rightarrow x=0$

(xiii) $\frac{x+1}{2}+\frac{2}{x+1}=\frac{x+1}{3}+\frac{3}{x+1}-\frac{5}{6}$, $x \neq -1$

$\Rightarrow \frac{a}{2}+\frac{2}{a}=\frac{a}{3}+\frac{3}{a}-\frac{5}{6}$ (Let $x+1=a$)

$\Rightarrow \left(\frac{a}{2}-\frac{a}{3}\right)+\left(\frac{2}{a}-\frac{3}{a}\right)+\frac{5}{6}=0$

$\Rightarrow \frac{a}{6}-\frac{1}{a}+\frac{5}{6}=0$

$\Rightarrow \frac{a^2-6+5a}{6a}=0$

$\Rightarrow a^2+5a-6=0$

$\Rightarrow a^2+6a-1\cdot a-6=0$

$\Rightarrow (a+6)(a-1)=0$

$\Rightarrow$ either $a=-6$ or $a=1$

when, $a=-6$ ie, $x+1=-6$ or $x=-7$

when $a=1$ ie, $x+1=1$ or $x=0$

(xiv) $\frac{12x+17}{3x+1}-\frac{2x+15}{x+7}=3\frac{1}{5}$, $x \neq -1/3, -7$

$\Rightarrow \frac{4(3x+1)+13}{3x+1}-\frac{2(x+7)+1}{x+7}=\frac{16}{5}$

$\Rightarrow 4+\frac{13}{3x+1}-2-\frac{1}{x+7}=\frac{16}{5}$

$\Rightarrow \frac{13}{3x+1}-\frac{1}{x+7}=\frac{16}{5}-2$

$\Rightarrow \frac{13x+91-3x-1}{3x^2+22x+7}=\frac{6}{5}$

$\Rightarrow \frac{10x+90}{3x^2+22x+7}=\frac{6}{5}$

$\Rightarrow (5x+45)\cdot 5=3(3x^2+22x+7)$

$\Rightarrow 9x^2+66x+21=25x+225$

$\Rightarrow 9x^2+41x-204=0$

$\Rightarrow 9x^2+68x-27x-204=0$

$\Rightarrow x(9x+68)-3(9x+68)=0$

$\Rightarrow (9x+68)(x-3)=0 \Rightarrow$ either $(9x+68)=0$ or $(x-3)=0 \Rightarrow x=-\frac{68}{9}$ or 3

Solve:

xv) $\frac{x+3}{x-3} + 6\left(\frac{x-3}{x+3}\right) = 5$

$\Rightarrow a + \frac{6}{a} = 5$ [assuming $\frac{x+3}{x-3} = a$]

$\Rightarrow a^2 - 5a + 6 = 0$

$\Rightarrow (a-3)(a-2) = 0$

$\Rightarrow$ either $a=3$ or $a=2$

if $a=3 \Rightarrow \frac{x+3}{x-3} = 3 \Rightarrow x+3 = 3x-9 \Rightarrow 2x = 12 \Rightarrow x = 6$

if $a=2 \Rightarrow \frac{x+3}{x-3} = 2 \Rightarrow x+3 = 2x-6 \Rightarrow x = 9$

Solve.

xvi) $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$, $x \neq 0, -(a+b)$

$\Rightarrow \frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$

$\Rightarrow \frac{x-a-b-x}{x(a+b+x)} = \frac{a+b}{ab}$

$\Rightarrow \frac{-(a+b)}{ax+bx+x^2} = \frac{(a+b)}{ab}$

$\Rightarrow x^2 + ax + bx = -ab$

$\Rightarrow x^2 + ax + bx + ab = 0$

$\Rightarrow x(x+a) + b(x+a) = 0$

$\Rightarrow (x+a)(x+b) = 0$

$\Rightarrow$ either $x = -a$ or $x = -b$

xvii) $\left(\frac{x+a}{x-a}\right)^2 - 5\left(\frac{x+a}{x-a}\right) + 6 = 0$, $x \neq a$

$\Rightarrow m^2 - 5m + 6 = 0$ (assuming $\frac{x+a}{x-a} = m$)

$\Rightarrow (m-3)(m-2) = 0$

$\Rightarrow$ either $m=3$ or $m=2$

when, $m=3$ then $\frac{x+a}{x-a} = 3 \Rightarrow x+a = 3x-3a \Rightarrow -2x = -4a \Rightarrow x = 2a$

when, $m=2$ then $\frac{x+a}{x-a} = 2 \Rightarrow x+a = 2x-2a \Rightarrow -x = -3a \Rightarrow x = 3a$

Solve:

xviii) $\frac{1}{x}-\frac{1}{x+b}=\frac{1}{a}-\frac{1}{a+b}$, $x\neq 0,-b$

$\Rightarrow \frac{x+b-x}{x^2+bx}=\frac{a+b-a}{a^2+ab}$

$\Rightarrow \frac{b}{x^2+bx}=\frac{b}{a^2+ab}$

$\Rightarrow x^2+bx=a^2+ba$

$\Rightarrow x^2+bx-a^2-ab=0$

$\Rightarrow (x+a)(x-a)+b(x-a)=0$

$\Rightarrow (x+a+b)(x-a)=0$

$\Rightarrow$ either $x=a$ or $x=-(a+b)$

Solve:

xix) $\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}+\frac{1}{(x-3)(x-4)}=\frac{1}{6}$, $x\neq 1,2,3,4$

$\Rightarrow \frac{(x-1)-(x-2)}{(x-1)(x-2)}+\frac{(x-2)-(x-3)}{(x-2)(x-3)}+\frac{(x-3)-(x-4)}{(x-3)(x-4)}=\frac{1}{6}$

$\Rightarrow \frac{1}{(x-2)}-\frac{1}{(x-1)}+\frac{1}{(x-3)}-\frac{1}{(x-2)}+\frac{1}{(x-4)}-\frac{1}{(x-3)}=\frac{1}{6}$

$\Rightarrow \frac{1}{x-4}-\frac{1}{x-1}=\frac{1}{6}$

$\Rightarrow \frac{x-1-x+4}{x^2-5x+4}=\frac{1}{6}$

$\Rightarrow x^2-5x+4=3\times 6$

$\Rightarrow x^2-5x+4-18=0$

$\Rightarrow x^2-5x-14=0$

$\Rightarrow (x-7)(x+2)=0$

$\Rightarrow$ either $x=7$ or $x=-2$.

xx) $\frac{a}{x-a}+\frac{b}{x-b}=\frac{2c}{x-c}$, $x\neq a,b,c$

$\Rightarrow \frac{a}{x-a}-\frac{c}{x-c}=\frac{c}{x-c}-\frac{b}{x-b}$

$\Rightarrow \frac{ax-ac-cx+ca}{(x-a)(x-c)}=\frac{cx-bc-bx+bc}{(x-b)(x-c)}$ (since $x-c\neq 0$)

$\Rightarrow x\frac{(a-c)}{x-a}-x\frac{(c-b)}{x-b}=0$

$\Rightarrow x\left(\frac{a-c}{x-a}-\frac{c-b}{x-b}\right)=0$

either, $x=0$

or, $\frac{a-c}{x-a} - \frac{c-b}{x-b} = 0$

$\Rightarrow \frac{ax-ab-cx+bc-cx+bx+ac-ab}{(x-a)(x-b)} = 0$

$\Rightarrow ax+bx-2cx-2ab+bc+ca = 0$

$\Rightarrow x(a+b-2c) - 2ab+bc+ca = 0$

$\Rightarrow x = \frac{2ab-bc-ca}{a+b-2c}$

Therefore the solution is, $x=0, \frac{2ab-bc-ca}{a+b-2c}$

xxi) $x^2-(\sqrt{3}+2)x + 2\sqrt{3} = 0$

$\Rightarrow x^2-\sqrt{3}x-2x+2\sqrt{3} = 0$

$\Rightarrow x(x-\sqrt{3}) - 2(x-\sqrt{3}) = 0$

$\Rightarrow (x-\sqrt{3})(x-2) = 0$

$\Rightarrow$ either $x-\sqrt{3}=0$ or $x-2=0$

$\Rightarrow$ either $x=\sqrt{3}$ or $x=2$.