Question 1:

Determine whether each of the following relations are reflexive, symmetric and transitive:

(i)Relation R in the set $A = \{1, 2, 3\ldots13, 14\}$ defined as

$R = \{(x, y): 3x - y = 0\}$

(ii) Relation R in the set **N** of natural numbers defined as

$R = \{(x, y): y = x + 5 \text{ and } x < 4\}$

(iii) Relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ as

$R = \{(x, y): y \text{ is divisible by } x\}$

(iv) Relation R in the set **Z** of all integers defined as

$R = \{(x, y): x - y \text{ is as integer}\}$

(v) Relation R in the set A of human beings in a town at a particular time given by

(a) $R = \{(x, y): x \text{ and } y \text{ work at the same place}\}$

(b) $R = \{(x, y): x \text{ and } y \text{ live in the same locality}\}$

(c) $R = \{(x, y): x \text{ is exactly 7 cm taller than } y\}$

(d) $R = \{(x, y): x \text{ is wife of } y\}$

(e) $R = \{(x, y): x \text{ is father of } y\}$

Answer

(i) $A = \{1, 2, 3 \ldots 13, 14\}$

$R = \{(x, y): 3x - y = 0\}$

$\therefore R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$

R is not reflexive since $(1, 1), (2, 2) \ldots (14, 14) \notin R$.

Also, R is not symmetric as $(1, 3) \in R$, but $(3, 1) \notin R$. $[3(3) - 1 \neq 0]$

Also, R is not transitive as $(1, 3), (3, 9) \in R$, but $(1, 9) \notin R$.

$[3(1) - 9 \neq 0]$

Hence, R is neither reflexive, nor symmetric, nor transitive.

(ii) $R = \{(x, y): y = x + 5 \text{ and } x < 4\} = \{(1, 6), (2, 7), (3, 8)\}$

It is seen that $(1, 1) \notin R$.

$\therefore R$ is not reflexive.

$(1, 6) \in R$

But,

(1, 6) ∉ R.

∴R is not symmetric.

Now, since there is no pair in R such that (x, y) and (y, z) ∈R, then (x, z) cannot bel

to R.

∴ R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

(iii) $A = \{1, 2, 3, 4, 5, 6\}$

R = $\{(x, y)$: y is divisible by $x\}$

We know that any number (x) is divisible by itself.

⇒ (x, x) ∈R

∴R is reflexive.

Now,

(2, 4) ∈R [as 4 is divisible by 2]

But,

(4, 2) ∉ R. [as 2 is not divisible by 4]

∴R is not symmetric.

Let (x, y), (y, z) ∈ R. Then, y is divisible by x and z is divisible by y.

∴z is divisible by x.

⇒ (x, z) ∈R

∴R is transitive.

Hence, R is reflexive and transitive but not symmetric.

(iv) R = $\{(x, y)$: $x - y$ is an integer$\}$

Now, for every $x \in$ **Z**, (x, x) ∈R as $x - x = 0$ is an integer.

∴R is reflexive.

Now, for every $x, y \in$ **Z** if $(x, y) \in$ R, then $x - y$ is an integer.

⇒ $-(x - y)$ is also an integer.

⇒ $(y - x)$ is an integer.

∴ $(y, x) \in$ R

∴R is symmetric.

Now,

Let (x, y) and (y, z) ∈R, where $x, y, z \in$ **Z**.

⇒ $(x - y)$ and $(y - z)$ are integers.

⇒ $x - z = (x - y) + (y - z)$ is an integer.

$\therefore$ $(x, z) \in R$

$\therefore$R is transitive.

Hence, R is reflexive, symmetric, and transitive.

(v) (a) R = $\{(x, y)$: x and y work at the same place$\}$

$\Rightarrow (x, x) \in R$

$\therefore$ R is reflexive.

If $(x, y) \in R$, then x and y work at the same place.

$\Rightarrow$ y and x work at the same place.

$\Rightarrow (y, x) \in R$.

$\therefore$R is symmetric.

Now, let $(x, y), (y, z) \in R$

$\Rightarrow$ x and y work at the same place and y and z work at the same place.

$\Rightarrow$ x and z work at the same place.

$\Rightarrow (x, z) \in R$

$\therefore$ R is transitive.

Hence, R is reflexive, symmetric, and transitive.

(b) R = $\{(x, y)$: x and y live in the same locality$\}$

Clearly $(x, x) \in R$ as x and x is the same human being.

$\therefore$ R is reflexive.

If $(x, y) \in R$, then x and y live in the same locality.

$\Rightarrow$ y and x live in the same locality.

$\Rightarrow (y, x) \in R$

$\therefore$R is symmetric.

Now, let $(x, y) \in R$ and $(y, z) \in R$.

$\Rightarrow$ x and y live in the same locality and y and z live in the same locality.

$\Rightarrow$ x and z live in the same locality.

$\Rightarrow (x, z) \in R$

$\therefore$ R is transitive.

Hence, R is reflexive, symmetric, and transitive.

(c) R = $\{(x, y)$: x is exactly 7 cm taller than $y\}$

Now,

$(x, x) \notin R$

Since human being x cannot be taller than himself.

∴R is not reflexive.

Now, let (x, y) ∈R.

⇒ x is exactly 7 cm taller than y.

Then, y is not taller than x.

∴ $(y, x) \notin$R

Indeed if x is exactly 7 cm taller than y, then y is exactly 7 cm shorter than x.

∴R is not symmetric.

Now,

Let $(x, y), (y, z) \in$ R.

⇒ x is exactly 7 cm taller than y and y is exactly 7 cm taller than z.

⇒ x is exactly 14 cm taller than z .

∴$(x, z) \notin$R

∴ R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

(d) R = $\{(x, y)$: x is the wife of $y\}$

Now,

$(x, x) \notin$ R

Since x cannot be the wife of herself.

∴R is not reflexive.

Now, let $(x, y) \in$ R

⇒ x is the wife of y.

Clearly y is not the wife of x.

∴$(y, x) \notin$ R

Indeed if x is the wife of y, then y is the husband of x.

∴ R is not transitive.

Let $(x, y), (y, z) \in$ R

⇒ x is the wife of y and y is the wife of z.

This case is not possible. Also, this does not imply that x is the wife of z.

∴$(x, z) \notin$ R

∴R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

(e) R = $\{(x, y)$: x is the father of $y\}$

$(x, x) \notin$ R

As x cannot be the father of himself.

$\therefore$R is not reflexive.

Now, let $(x, y) \in R$.

$\Rightarrow x$ is the father of y.

$\Rightarrow y$ cannot be the father of y.

Indeed, y is the son or the daughter of y.

$\therefore (y, x) \notin R$

$\therefore$ R is not symmetric.

Now, let $(x, y) \in R$ and $(y, z) \in R$.

$\Rightarrow x$ is the father of y and y is the father of z.

$\Rightarrow x$ is not the father of z.

Indeed x is the grandfather of z.

$\therefore (x, z) \notin R$

$\therefore$R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.