Right Circular Cone

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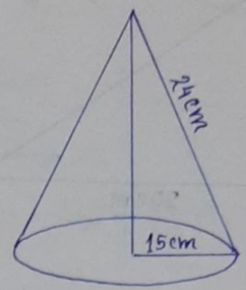
Letus work out 16 (Pg 227)

1. The right circular cone's,

radius $= r = 15$ cm

slant height $= l = 24$ cm



Therefore, curved surface area,

$= \pi r l$

$= \left(\frac{22}{7} \times 15 \times 24\right) cm^2$

$= \frac{7920}{7} cm^2$

$= 1131\frac{3}{7}$ sqcm.

Total surface area $= \pi r l + \pi r^2 = \pi r(l+r) = \frac{22}{7} \times 15 \times (24+15)$ sqcm

$= \frac{22 \times 15 \times 39}{7}$ sqcm

$= \frac{12870}{7}$ sqcm

$= 1838\frac{4}{7}$ sqcm.

2.i) Given cone's, height $= h = 2.4$ m

if r is the radius then, base area $= \pi r^2 = 1.54$ sqm (given).

So the cones valume $= \frac{1}{3} \times \pi r^2 \times h$

$= \left(\frac{1}{3} \times 1.54 \times 2.4\right)$ cubic metre

$= 1.232$ cubic metre.

ii) Given cone's slant height $= l = 17.5$m

base diameter $= 21$ m

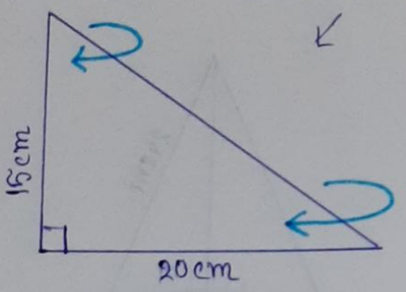
$\therefore$ base radius $= r = \frac{21}{2}$ m $= 10.5$ m.

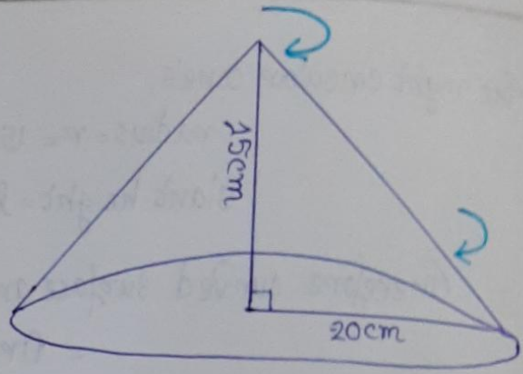
So, height $= h = \sqrt{l^2 - r^2} = \sqrt{(17.5)^2 - (10.5)^2} = \sqrt{306.25 - 110.25} = \sqrt{196} = 14$m

So, volume of the cone is $= \frac{1}{3}\pi r^2 h$

$= \left(\frac{1}{3} \times \frac{22}{\cancel{7}} \times 10.5 \times \overset{3.5}{\cancel{10.5}} \times \overset{2}{\cancel{14}}\right) m^3$

3\. Right angled triangle drawn by Amina





When she was revolving the triangle she had taken 15cm side as axis

So, the newly formed cone's,

height $= h = 15cm$

base-radius $= r = 20$ cm

$\therefore$ slant height $= l = \sqrt{h^2+r^2} = \sqrt{15^2+20^2} = \sqrt{625} = 25cm$

So, curved surface area $= \pi r l = \left(\frac{22}{7} \times 20 \times 25\right) cm^2 = \frac{11000}{7}$ sqcm

$= 1571\frac{3}{7}$ sqcm.

Total surface area $= \pi r l + \pi r^2$

$= \pi r(l+r) = \frac{22}{7} \times 20 \times (20+25) = \frac{22 \times 20 \times 45}{7}$ sqcm

$= \frac{19800}{7}$ sqcm

$= 2828\frac{4}{7}$ sqcm

volume

$= \frac{1}{3}\pi r^2 h$

$= \left(\frac{1}{\cancel{3}} \times \frac{22}{7} \times 20 \times 20 \times \overset{5}{\cancel{15}}\right)$ cubic cm

$= \frac{44000}{7}$ cm^3

$= 6285\frac{5}{7}$ cm^3.

4. Given cone's height = $h = 6cm$

and slant height = $l = 10cm$

So base radius = $r = \sqrt{l^2 - h^2} = \sqrt{10^2 - 6^2} = 8cm$

Total surface area = $\pi r(l + r)$

$= \frac{22}{7} \times 8 \times (8 + 10) = \frac{3168}{7}$ sqcm = $452\frac{4}{7}$ sqcm.

volume of the cone = $\frac{1}{3}\pi r^2 h$

$= \left(\frac{1}{3} \times \frac{22}{7} \times 8 \times 8 \times 6\right)$ cubic cm

$= \frac{2816}{7}$ cubic cm = $402\frac{2}{7}$ cubic cm.

5. Given cone's,

height = $h = 12$ cm

radius is not given, Let us assume the radius is = r

then the volume of the cone is = $\frac{1}{3}\pi r^2 h = \left(\frac{1}{3}\pi r^2 \times 12\right)$ cubic cm

But it is given, that the volume is 100π cm^3

∴ According to the problem,

$\frac{1}{\cancel{3}}\cancel{\pi} r^2 \times \cancel{12}^{4} = \cancel{100}^{25} \cancel{\pi}$

$\Rightarrow r^2 = 25$

$\Rightarrow r = 5$ $(\because r > 0)$

So the radius is 5cm

now, slant height $l = \sqrt{r^2 + h^2}$

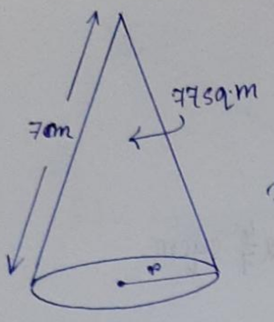
$= \sqrt{5^2 + 12^2}$ cm

$= \sqrt{25 + 144}$ cm

$= \sqrt{169}$ cm

$= 13$ cm $(\because l > 0)$

6.



The given cone's,

slant height $= l = 7\,m$

77 sqm tripal used to make the conical tent, so the conical tent's curved surface area $= 77$ sqm.

Let us assume the radius be $= r$ m.

so, curved surface area $= \pi r l$

According to the problem,

$\pi r l = 77$

$\Rightarrow \frac{\cancel{22}^{2}}{\cancel{7}} \times r \times \cancel{7} = \cancel{77}^{7}$

$\Rightarrow r = 7/2$

So base area of the tent is $= \pi r^2 = \left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right)$ sqm $= \frac{77}{2} = 38.5$ sqm.

7. Given cone's,

base diameter $= 21\,m$

$\therefore$ base radius $= r = \frac{21}{2}\,m$ and height $= h = 14\,m$

therefore, slant height $= l = \sqrt{r^2 + h^2} = \sqrt{\left(\frac{21}{2}\right)^2 + (14)^2}\ m$

$= \sqrt{\frac{441}{4} + 196}\ m$

$= \sqrt{\frac{441 + 784}{4}}\ m$

$= \sqrt{\frac{1225}{4}}\ m = \frac{35}{2}\,m$

So, curved surface area $= \pi r l$

$= \left(\frac{22}{7} \times \frac{21}{2} \times \frac{35}{2}\right)$ sqm

$\therefore$ The expenditure to color curved surface area is ₹ $=$ (curved area $\times 1.50$)

$=$ ₹ $\left(\frac{22}{7} \times \frac{21}{2} \times \frac{35}{2} \times \frac{150}{100}\right)$

$=$ ₹ 866.25

8. The conical wooden toy's,

base diameter = 10 cm, therefore base radius $= r = \frac{10}{2} = 5$ cm

Let the slant height is $= l$

then curved surface area $= \pi r l = \left(\frac{22}{7} \times 5 \times l\right)$ sq cm.

Since the cost of polishing curved surface area per cm^2 is ₹ 2.10 so, total cost of polishing curved surface area is,

$= \text{curved area} \times ₹ 2.10$

$= ₹ \left(\frac{22}{7} \times 5 \times l \times \frac{210}{100}\right)$

According to the problem,

$₹ \frac{\cancel{22}}{\cancel{7}} \times \cancel{5} \times l \times \frac{\cancel{210}^{3}}{\cancel{100}} = 4\cancel{29}\ 39\ 13$

$\Rightarrow l = 13$

So the slant height is 13 cm

So, height $h = \sqrt{l^2 - r^2} = \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = \sqrt{144} = 12$ $(\because h > 0)$

So, height of the cone is 12 cm. ______ (Ans).

now, quantity of wood required to make the toy

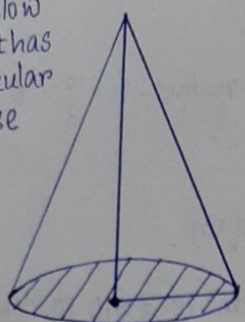
$= \text{volume of the toy}$

$= \frac{1}{3}\pi r^2 h$

$= \left(\frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 12\right)\ cm^3$

$= 2200/7 = 314\frac{2}{7}\ cm^3$

9. hollow but has circular base



The quanlity of iron sheet to make the conical boya is $= 75\frac{3}{7} = \frac{528}{7} m^2$

So total surface area ie, sum of curved and plane surface area is $= \frac{528}{7} m^2$

Given conical boya's, slant height $= l = 5m$

Let base radius be $= r$

So, Total surface area is,

$\pi r(r+l) = \frac{22}{7} \times r \times (r+5) \; m^2$

According to the problem,

$\frac{\cancel{22}}{\cancel{7}} \times r \times (r+5) = \frac{\cancel{528}\ 24}{\cancel{7}}$

$\Rightarrow r(r+5) = 24$

$\Rightarrow r^2 + 5r - 24 = 0 \Rightarrow (r+8)(r-3) = 0 \Rightarrow r = -8 \text{ or } 3$

Since $r > 0$, $r = 3$

$\therefore$ radius $= 3m$

$\therefore$ height $= h = \sqrt{l^2 - r^2} = \sqrt{5^2 - 3^2} = \sqrt{25-9} = \sqrt{16} = 4m$ $(\because h > 0)$

(Ans).

now volume of air in the boya

$=$ volume of the boya

$= \frac{1}{3}\pi r^2 h$

$= \left(\frac{1}{\cancel{3}} \times \frac{22}{7} \times \cancel{3} \times 3 \times 4\right) m^3 = \frac{264}{7} = 37\frac{5}{7} m^3$ (Ans).

$= \cancel{\frac{254}{7}} = \cancel{36\frac{2}{7}}$

Now curved surface area $= \pi r l = \left(\frac{22}{7} \times 3 \times 5\right) m^2$

Total cost of coloring curved surface

area $= ₹\left(\frac{22}{7} \times 3 \times 5 \times \overset{0.40}{2.80}\right)$

$= ₹\ 132$

Base area $= \pi r^2 = \left(\frac{22}{7} \times 3 \times 3\right) m^2$ and cost of coloring base $= ₹\left(\frac{22}{7} \times 3 \times 3 \times 2.80\right)$

$= ₹\ 79.20$

$\therefore$ Cost of coloring whole surface $= ₹(132 + 79.20) = ₹\ 211.20$

10. Each person needs = $4m^2$ space in the base.

so, 11 persons need $= (11\times4)\,m^2 = 44\,m^2$ space in base.

So, clearly the base area of the tent should be $= 44\,m^2$.

Let r be the base radius of the the tent, so base area $= \pi r^2$

$\therefore \pi r^2 = 44$ ------ (i)

Now, each person needs $= 20\,m^3$ air

So, 11 persons need $= (20\times11) = 220\,m^3$ air.

So, clearly the volume of the tent should be $= 220\,m^3$.

Let us assume, for that volume, the necessary height is $= h\,m$.

then volume of the tent $= \frac{1}{3}\pi r^2 h$

According to the problem,

$$\frac{1}{3}\pi r^2 h = 220$$

$$\Rightarrow \frac{1}{3}\times \cancel{44}\times h = \cancel{220}^{5}$$ (putting value of πr^2 from (i))

$$\Rightarrow h = 15.$$

$\therefore$ height of the tent put up exactly for 11 persons is $= 15\,m$.

11. To wrap up the outer surface of the coronet, total expenditure will be ₹57.75 at rate of 10 paise per cm^2

So curved surface area is $= \left(\frac{₹57.75}{10p.}\right) cm^2 = \left(\frac{5775p}{10p}\right) cm^2 = 577.5\,cm^2$

outer radius of the coronet $= r = \frac{21}{2}$ cm

let slant height $= l$

$\therefore$ curved surface area $= \pi r l = \left(\frac{22}{7}\times\frac{21}{2}\times l\right) cm^2$

According to the prob,

$$\frac{22}{7}\times\frac{21}{2}\times l = 577.5$$

$\Rightarrow \frac{\cancel{22}}{\cancel{7}} \times \frac{\cancel{21}}{\cancel{2}} \times l = \frac{\cancel{5775}\ \cancel{525}\ 175}{10}$

$\Rightarrow l = 17.5$

So the slant height is $= 17.5$ cm (Ans).

then height is, $h = \sqrt{l^2 - r^2}$

$= \sqrt{17.5^2 - 10.5^2}$

$= \sqrt{306.25 - 110.25} = \sqrt{196} = 14 \quad (\because h > 0)$

So, height is 14 cm (Ans).

12. The right circular conical heap of wheat's,

base diameter $= 9$m

$\therefore$ base radius $= r = \frac{9}{2}$ m $= 4.5$ m

and it is given the height is $= h = 3.5$ m

Total volume of the heap of wheat,

$= \frac{1}{3}\pi r^2 h = \left(\frac{1}{3} \times \frac{22}{7} \times 4.5 \times 4.5 \times 3.5\right) m^3 = 74.25\ m^3$

(Ans).

Slant height $= l = \sqrt{r^2 + h^2}$

$= \sqrt{4.5^2 + 3.5^2} = \sqrt{32.50} = \sqrt{\frac{\cancel{3250}\ \cancel{650}\ 130}{\cancel{100}\ \cancel{20}\ 4}} = \sqrt{\frac{130}{4}}$ m

$= \frac{\sqrt{130}}{2}$ m.

curved surface area

$= \pi r l$

$= 3.14 \times 4.5 \times \frac{\sqrt{130}}{2} = \left(3.14 \times 4.5 \times \frac{11.4}{2}\right) m^2 = 80.541\ m^2$

So, the plastic sheet, required to cover up the heap of wheat is $= 80.541\ m^2$.