Trigonometric Ratios of Positive Acute Angle :-

1) If $a\cos x + b\sin x = c$. Then find the value of $(a\sin x - b\cos x)^2$

Ans:- We have, $(a\cos x + b\sin x)^2 + (a\sin x - b\cos x)^2$

$= a^2 + b^2$

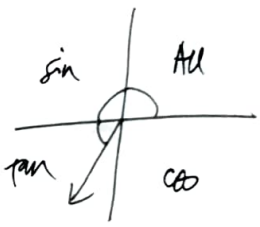
$\Rightarrow c^2 + (a\sin x - b\cos x)^2 = a^2 + b^2$

$\Rightarrow (a\sin x - b\cos x)^2 = a^2 + b^2 - c^2$

2) Find the value of $\cos 5\pi$.

Ans:- We have $\cos 5\pi = \cos(\pi + 4\pi) = \cos\pi$

$= -1$ (Ans)



3) In a triangle ABC, $\operatorname{cosec} A(\sin B\cos C + \cos B\sin C)$ is equal to ?

Ans: $\operatorname{cosec} A(\sin B\cos C + \cos B\sin C)$

$= \operatorname{cosec} A \sin(B + C)$

$= \operatorname{cosec} A \sin(180^\circ - A)$ $[\because (A + B + C) = 180^\circ]$

$= \operatorname{cosec} A \cdot \sin A = 1$ (Ans)

4) Find the value of $\cos 180^\circ$.

Ans:- Theoritically we have $\cos(90^\circ + \theta) = -\sin\theta$.

$\Rightarrow \cos(90^\circ + 90^\circ) = -\sin 90^\circ$

$\Rightarrow \cos 180^\circ = -1$ $[\because \sin 90^\circ = 1]$

(Ans)

5) Find the value of $\cos 15^\circ + \sin 15^\circ$.

Ans:- $\sin 15^\circ + \cos 15^\circ$

$= \sin 15^\circ + \cos(90^\circ - 15^\circ)$ $[\because \cos(90^\circ - \theta) = \sin\theta]$

$= \sin 15^\circ + \sin 15^\circ$

$= 2\sin 45^\circ \cos 30^\circ$

$= 2\left(\frac{1}{\sqrt{2}}\right) \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

6) Find the value of $\cos 420^\circ$ —

Ans:- $\cos 420^\circ = \cos(360^\circ + 60^\circ)$

$= \cos 60^\circ$

$= \frac{1}{2}$

7) Find the least value of $\cos^2\theta + \sec^2\theta$. —

Ans:- $a\cos^2\theta + b\sec^2\theta$ has the least value of $2\sqrt{ab}$

Now, $\cos^2\theta + \sec^2\theta$ the value is $a=1, b=1$.

least value $2\sqrt{1\times 1} = 2\times 1 = 2$.

8) If $x > 0$ then the value of $f(x) = -3 \times \cos\sqrt{(3+x+x^2)}$. lie in which interval?

Ans:- Given $x > 0$, then $3 + x + x^2 > 0$

Now, $-1 \le \cos\sqrt{3+x+x^2} \le 1 \quad \{\because -1 \le \cos x \le 1\}$

$\Rightarrow 3 \ge -3 \times \cos\sqrt{(3+x+x^2)} \ge -3$

(Multiply by -3)

$\Rightarrow -3 \le f(x) \le 3$

$\Rightarrow f(x) \in \{-3, 3\}$

9) If $\tan\theta + \cot\theta = 2$, then what is the value of $\tan^{100}\theta + \cot^{100}\theta$

Ans:- Given $\tan\theta + \cot\theta = 2$.

Put $\theta = 45^\circ$, then the above equation will satisfy as

$1 + 1 = 2$.

$\theta = 45^\circ$

$= \tan^{100} 45^\circ + \cot^{100} 45^\circ$

$= 1^{100} + 1^{100} = 2$

10) If $4\cos^2\theta - 1 = 0$, then find the value of $\tan(\theta - 15^\circ)$

Ans:- If $4\cos^2\theta - 1 = 0$

$\Rightarrow 4\cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1/4$

$\Rightarrow \cos\theta = 1/2 \Rightarrow \theta = 1/2\, 60^\circ$

$\therefore \tan(\theta - 15^\circ) = \tan(60^\circ - 15^\circ)$

$= \tan 45^\circ = 1$

11) If $r\sin\theta = 1$ & $r\cos\theta = \sqrt{3}$, find the value of $(\sqrt{3}\tan\theta + 1)$

Ans:- We have $\frac{r\sin\theta}{r\cos\theta} = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = 1/\sqrt{3}$

$\Rightarrow \theta = 30^\circ$

$\therefore (\sqrt{3}\tan 30^\circ + 1) = \sqrt{3} \cdot \frac{1}{\sqrt{3}} + 1 = 1 + 1 = 2.$

12) What is the value of $\tan^2\theta - \sec^2\theta$?

Ans:- $\tan^2\theta - \sec^2\theta$

$= \frac{\sin^2\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta}$

$= \frac{\sin^2\theta - 1}{\cos^2\theta} = -\frac{\cos^2\theta}{\cos^2\theta} = -1$