Concepts of Measurements of Angle

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Let us work out 20

i) $832' = 13°52'$

[since $1° = 60'$
$1' = 60''$]

$60\overline{)832'}(13°$
60
232
180
$52'$

ii) $6312''$
$= 105'12''$
$= 1°45'12''$

$60\overline{)6312''}(105'$
60
312
300
$12''$

$60\overline{)105'}(1°$
60
$45'$

iii) $375'' = 6'15''$

$60\overline{)375''}(6'$
360
$15''$

iv) $27\frac{1}{12}° = 27°\left(\frac{1}{12}\times 60\right)' = 27°5'$

v) $72.04° = 72° + \frac{4}{100}° = 72°\left(\frac{4}{100}\times 60\right)' = 72°(2.4)' = 72°2'\left(\frac{4}{10}\times 60\right)'' = 72°2'24''$

Note: In sexagesimal system units of angles are degree(°), minute('), sec('')
In circular system units of angles are radian(c)

Relation between sexagesimal and circular system units.

$180° = \pi^c$ or simply $180° = \pi$ (we usually don't write "c")

2) **Process one**

$180° = \pi$
$\Rightarrow 1° = \frac{\pi}{180}$
$\Rightarrow 60° = \left(\frac{\pi}{180}\times 60\right) = \frac{\pi}{3}$

Process two

$60° = \left(60 \times \frac{\pi}{180}\right)$
$= \frac{\pi}{3}$

iii) $135^\circ = 135 \times \frac{\pi}{180} = \frac{3\pi}{4}$

iii) $-150^\circ = -\left(150^\circ \times \frac{\pi}{180}\right) = -\frac{5\pi}{6}$

iv) $72^\circ = \left(72 \times \frac{\pi}{180}\right) = \frac{2\pi}{5}$

v) $22^\circ 30'$

$= 22^\circ + \left(\frac{30}{60}\right)^\circ = 22^\circ + \frac{1}{2}^\circ = \frac{45^\circ}{2} = \left(\frac{45}{2} \times \frac{\pi}{180}\right) = \frac{\pi}{2\times 4} = \frac{\pi}{8}$

vi) $-62^\circ 30'$

$= -62^\circ + \left(-\frac{30}{60}\right)^\circ = -62^\circ - \frac{1}{2}^\circ = \frac{-135^\circ}{2} = -\frac{135^\circ}{2} \times \frac{\pi}{180} = -\left(\frac{3\pi}{2\times 4}\right) = -\frac{3\pi}{8}$

vii) $52^\circ 52' 30''$

$= 52^\circ + \frac{52^\circ}{60} + \frac{30^\circ}{3600}$

$= 52^\circ + \frac{13^\circ}{15} + \frac{1^\circ}{120}$

$= \left(\frac{6240+104+1}{120}\right)^\circ = \frac{6345^\circ}{120} = \left(\frac{6345}{120} \times \frac{\pi}{180}\right) = \frac{47\pi}{160}$

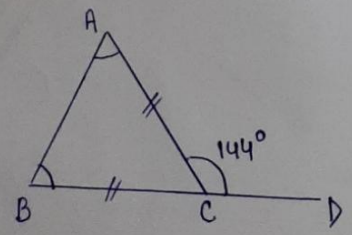
141 47
423
1269
24 8
60 20

viii) $40^\circ 16' 24''$

$= 40^\circ + \frac{16^\circ}{60} + \frac{24^\circ}{3600}$

$= 40^\circ + \frac{4^\circ}{15} + \frac{1^\circ}{150} = \frac{6000^\circ + 40^\circ + 1^\circ}{150} = \frac{6041^\circ}{150} = \frac{6041}{150} \times \frac{\pi}{180} = \frac{6041\pi}{27000}$.

3.



$\angle ACB = 180^\circ - \angle ACD = 180^\circ - 144^\circ = 36^\circ = 36^\circ \times \frac{\pi}{180} = \frac{\pi}{5}$

Since, $AC = BC$

therefore $\angle CAB = \angle CBA = \frac{180^\circ - \angle ACB}{2}$

$= \frac{180^\circ - 36^\circ}{2} = \frac{144^\circ}{2}$

$= 72^\circ$

$= 72 \times \frac{\pi}{180}$

$= \frac{2\pi}{5}$

So, $\angle ABC = \frac{\angle BAC}{\angle ACB} = \frac{2\pi}{5}$

$\angle ACB = \frac{\pi}{5}$.

4. In the right angled triangle,

one angle is $= 90° = \pi/2$.

Let one of the acute angles is $= x$

then the other angle is $= x + \frac{2\pi}{5}$.

now sum of three angles in a triangle is $= \pi$

So,

$$\frac{\pi}{2} + x + \left(x + \frac{2\pi}{5}\right) = \pi$$

$$\Rightarrow 2x + \frac{5\pi + 4\pi}{10} = \pi$$

$$\Rightarrow 2x + \frac{9\pi}{10} = \pi$$

$$\Rightarrow 2x = \pi - \frac{9\pi}{10}$$

$$\Rightarrow 2x = \frac{10\pi - 9\pi}{10}$$

$$\Rightarrow 2x = \frac{\pi}{10}$$

$$\Rightarrow x = \frac{\pi}{20}.$$

So, sexagesimal values of other angles are,

$$\frac{\pi}{20} = \left(\frac{}{20} \times 180\right)^\circ = 9^\circ$$

and $\frac{\pi}{20} + \frac{2\pi}{5} = \frac{\pi + 8\pi}{20} = \frac{9\pi}{20} = \left(\frac{9}{20} \times 180\right)^\circ = 81^\circ$

5. Value of two angles are,

65° and $\frac{\pi}{12} = \left(\frac{180}{12}\right)^\circ = 15^\circ$

So, sexagesimal value of third angle $= 180^\circ - (65^\circ + 15^\circ) = 100^\circ$

and circular value $= 100 \times \frac{\pi}{180} = \frac{5\pi}{9}$

6. Let two angles be x and y in sexagesimal system,

So, $x+y=135^\circ$ ——— (i)

their difference $\frac{\pi}{12}=\left(\frac{180}{12}\right)^\circ=15^\circ$

So, $x-y=15^\circ$ ——— (ii)

adding (i) and (ii)

$$\begin{array}{r} x+y=135^\circ \\ x-y=15^\circ \\ \hline 2x=150^\circ \end{array}$$

$\Rightarrow x=75^\circ$

from (i) $y=135^\circ-75^\circ=60^\circ$

now, $75^\circ=75\times\frac{\pi}{180}=\frac{5\pi}{12}$

$60^\circ=60\times\frac{\pi}{180}=\frac{\pi}{3}$

7. Sum of three angles in a triangle is π (in circular system)

Let the three angles be $2x$, $3x$ and $4x$.

So, $2x+3x+4x=\pi$

$\Rightarrow x=\frac{\pi}{9}$

So, greatest angle $=4x=\frac{4\pi}{9}$.

Second method.

ratio of three angles $=2:3:4$

sum of three angles $=\pi$

so, greatest angle $=\left(\frac{4}{2+3+4}\times\pi\right)=\frac{4\pi}{9}$.

8. Circle's, radius = $r = 28$ cm

circumference $= 2\pi r = \left(2 \times \frac{22}{7} \times 28\right)$ cm

So, angle subtended by $\left(2 \times \frac{22}{7} \times 28\right)$ cm circumference $= 2\pi$ $\quad [\because 2\pi = 360^\circ]$

by 1 cm arc $= 2\pi \times \frac{7}{2 \times 22 \times 28}$

angle subtended by $\frac{55}{10}$ cm arc $= \frac{2 \times 22}{7} \times \frac{7}{2 \times 22 \times 28} \times \frac{55}{10}$

$= \frac{55}{28 \times 10}$

$= \frac{55\pi}{4 \times 28 \times 10_2} \times \frac{7}{22_2}$

$= \frac{\pi}{16}$.

9. Let sexagesimal value of first angle x.

So, $\frac{x}{30^\circ} = \frac{5}{2}$

$\Rightarrow x = 30^\circ \times \frac{5}{2} = 75^\circ$

$75^\circ = 75 \times \frac{\pi}{180} = \frac{5\pi}{12}$ So the value of first angle in sexagesimal system is 75° and in circular system is $\frac{5\pi}{12}$

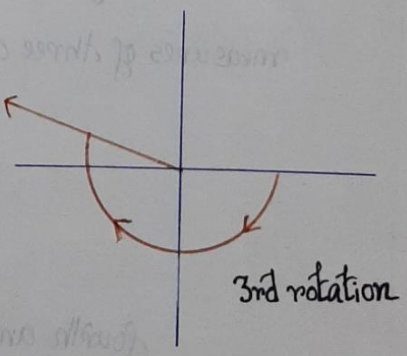
10. $-5\frac{1}{12}\pi = -\frac{61}{12}\pi = -\frac{61 \times 180^\circ}{12} = -915^\circ$

Complete revolution of ray $= 360^\circ$

$360)\ 915^\circ\ (2$

$\quad 720$

$\quad 195^\circ$



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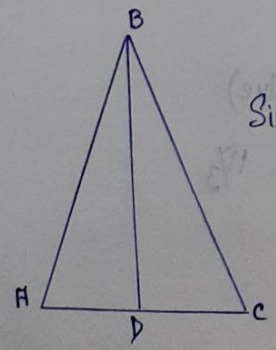


$\angle ABC = 45^\circ$

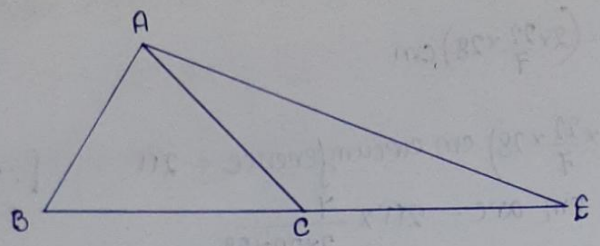
Since AB = AC thus $\angle BAC = \angle BCA$

Since AD is bisector of $\angle B$ so $\overset{BD}{AD} \perp AC$ (Since $\triangle ABC$ is isosceles)

now, $\angle ABD = \angle CBD = \frac{45^\circ}{2} = \left(\frac{45}{2} \times \frac{\pi}{180}\right) = \frac{\pi}{8}$

$\angle BAC = \angle BCA = \frac{(180 - 45)^\circ}{2} = \frac{135^\circ}{2} = \left(\frac{\overset{3}{135}}{2} \times \frac{\pi}{\underset{4}{180}}\right) = \frac{3\pi}{4}$

12.



in

Since ΔABC,

$AB = BC = CA$

$\angle ABC = \angle BAC = \angle ACB = \frac{180^\circ}{3} = 60^\circ$

So in ΔACE,

$\angle ACE = 180^\circ - \angle ACB = 180^\circ - 60^\circ = 120^\circ = 120 \times \frac{\pi}{180} = 2\pi/3$

Since, $BC = CE$ (according to drawing)

and $BC = AC$

So, $CE = AC$ So, ΔACE is isosceles,

$\angle CAE = \angle CEA = \frac{180^\circ - 120^\circ}{2} = \frac{60^\circ}{2} = 30^\circ = 30 \times \frac{\pi}{180} = \frac{\pi}{6}$

13. Sum of four angles of a quadrilateral, is 360°

measures of three angles in given quadrilateral are,

$\frac{\pi}{3} = 60^\circ$

$\frac{5\pi}{6} = \frac{5}{6} \times 180^\circ = 150^\circ$

and 90°

fourth angle is $= 360^\circ - (60^\circ + 150^\circ + 90^\circ)$

$= 360^\circ - 300^\circ$

$= 60^\circ$ (sexagesimal value)

circular value of fourth angle $= \left(60 \times \frac{\pi}{180}\right) = \pi/3$.